LETTERS TO THE EDITORS

SPACE SCIENCE

Errors in Orbital Predictions for Artificial Satellites of Earth

SEVERAL workers¹⁻³ have carefully investigated variations in the accelerations of artificial satellites of Earth, but no one seems to have published a systematic study of the errors in orbital predictions. The results of such work⁴ are presented in this communication. A stochastic model was constructed

of revolutions predicted, beginning at the centre of the smoothing interval. The smoothed rate of change of period is dP/dn (min./revolution). The root-meansquare prediction error, E (min.), includes the contributions of observational errors and drag fluctuations. The theoretical prediction error caused by observational errors alone is designated by E_{0} .

It is interesting to note that observational errors were the principal cause of errors in orbital predictions for only one of the cases shown in Fig. 1, that of Vanguard I with its perigee in darkness (winter

Table 1. ERRORS IN PREDICTION NEAR PEAK OF SUNSPOT CYCLE

Satellite	Dates	No. of predictions	-dP/dn (min./rev.)	N (rev.)	<i>E</i> ₀ (min.)	Actual (min.)	E Theo ret ical (min.)
Explorer IV	1958	8	$\begin{array}{c} 2 \cdot 15 \times 10^{-3} \\ 1 \cdot 32 \times 10^{-3} \\ 5 \cdot 5 \times 10^{-5} \\ 2 \cdot 1 \times 10^{-5} \\ 6 \cdot 5 \times 10^{-6} \\ 2 \cdot 2 \times 10^{-2} \end{array}$	165	0-024	3.2	3·3
Sputnik III	1958	7		220	0-01	3.3	1·9
Vanguard I	Autumn 1958	20		154	0-056	0.25	0·22
Vanguard I	Summer 1959	11		154	0-056	0.13	0·097
Vanguard I	Winter 1959–1960	7		154	0-056	0.062	0·061
Atlas-Score	Dec. 1958–Jan. 1959	1 *		271	0-3	67	74

* A single observation has no statistical significance. This case is included merely to show how large the error can be when the rate of change of period is large.

Table 2. ERRORS IN PREDICTION HALF-WAY BETWEEN SUNSPOT MAXIMUM AND MINIMUM

Satellite	Dates	No. of predictions	-dP/dn (min./rev.)	N (rev.)	E. (min.)	Actual (min.)	Theoretical (min.)
Tiros II Vanguard I	Dec. 1960–May 1961 Oct. 1960–May 1961	12 12	$\begin{array}{rrrr} 3.7 & imes 10^{-6} \ 7.4 & imes 10^{-6} \end{array}$	$250 \\ 150$	0·08 0·06	0·12 0·12	0-08 0-06
Transit III-B Echo I	Feb.–Mar. 1961 Oct–Dec. 1960	10 6	1.05×10^{-2} 6.8×10^{-4}	$\begin{array}{c} 22 \\ 145 \end{array}$	0·04 0·04	0·74 4·4	0.50 3.3

which takes into account fluctuations in atmospheric density, observational errors, and the correlations introduced by smoothing. Errors in actual predictions issued by the Vanguard Computing Center, National Aeronautics and Space Administration Computing Center, Smithsonian Astrophysical Observatory, and Naval Weapons Laboratory were also computed, for comparison with the theoretical model.

The actual and theoretical errors in orbital predic-tions are given in Tables 1 and 2 and Fig. 1. Table 1 contains the errors in 1-2-week predictions made near the peak of the sunspot cycle. Fig. 1 presents the same data graphically. Table 2 shows the errors in predictions half-way between sunspot maximum and sunspot minimum. In Tables 1 and 2, N is the number



1959-1960). In all the cases shown in Fig. 1, the prediction errors attributable to observational errors were smaller than the total error for Vanguard I in darkness. If the errors in the predictions had been caused only by observational errors, then the prediction errors would have been independent of the smoothed rate of change of period. A detailed discussion of the theory and the method of calculation is given in ref. 4.

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Priester, W., and Martin, H. A., Mitteilung der Universitäts-Sternwarte Bonn, No. 29 (1960).
Paetzold, H. K., A Proposal for a Self-Consistent Model of the Upper Atmosphere (Technische Hochschule, München, 1961).

⁴ Moe, K., TR-60-0000-09145 (Space Technology Laboratories, Inc., April 1960).

ASTROPHYSICS

Secular Variations of Short-lived Sunspots

LARGE secular variations have been found in shortlived sunspots as recorded in "Sunspots seen on one Day Only" of the Greenwich Photo-Heliographic Results, 1879–1957. The 11-yr. period often appears like a subsidiary perturbation.

A pertinent example is seen in Fig. 1, which shows the ratio K between the number of one-day spots at less than 65° from the central meridian, and the number of spots seen throughout two days or more. In Fig. 2 is plotted the number of one-day spots without umbra as a percentage of the total number.