can be regarded as the standard deviation of the distribution of $[(r + \Delta r) - r]$, assuming the two terms to be uncorrelated, and will be further referred to as σ_r. The constant will be called b.

Marriott[4] has shown that this equation leads to rather implausible consequences: (1) Although when the threshold is defined by a criterion such as 80 per cent of positive responses, ΔI will decrease as the area increases (causing σ_r to decrease), as Gregory[1] found, this will not be true when the criterion is 50 per cent positive responses, in which case threshold will be independent of changes in area. (2) With a criterion such as 20 per cent, ΔI decreases as the area decreases (σ_r increases). It might be added (3) that though the threshold will stay constant despite changes in area with a 50 per cent criterion, the false positive rate will decrease as the area increases. Consequently, Marriott rejects the approximations to Ricco's and Piper's laws for areal summation at the absolute threshold which Cane and Gregory derive from their assumptions.

The modification of the equation in a way suggested by the theory of signal detectability appears capable of avoiding these difficulties. In place of Gregory's assumption that a fixed difference between impulse-rates (or whatever central effects of the stimuli determine the responses) is required by the brain to establish discrimination, let us suppose that the discrimination mechanism operates to give maximum detection consistent with a limiting rate of false positive responses. In this case the value of C will not be constant, but must be defined as a function of σ_r. Let us therefore replace it by $m\sigma_r$, where the value of m determines the proportion of false positives. Thus, when $m = 2$, the false positive rate would be approximately equal to 2·5 per cent. The equation for the threshold can now be expressed as:

$$[(r + \Delta r) - r] = (m + b)\sigma_r \qquad (2)$$

Assuming that the distribution of $[(r + \Delta r) - r]$ is approximately normal, b is the normal deviate corresponding to the frequency of response required to define the threshold. When this is 50 per cent, $b = 0$ and the right-hand terms reduce to $m\sigma_r$. Thus the threshold is still directly proportional to σ_r, so that it will vary with area just as when it is defined by an 80 per cent requirement. This meets Marriott's first point. When the requirement is 20 per cent, the equation becomes $(r + \Delta r) - r = (m - b)\sigma_r$, where b is the normal deviate corresponding to 80 per cent negative responses. In this case too the threshold will decrease as σ_r decreases, that is, as the area increases, thus meeting the second objection. Moreover, so long as the response-rate required to define the threshold exceeds the false positive rate, $(m - b)$ will be positive.

Even though the threshold equation in the form above meets Marriott's objections, there is still reason to doubt the derivation of Ricco's and Piper's laws, based on the assumption that r is proportional to $\log(I + k)$, which is offered by Cane and Gregory, where k is a constant assumed by Gregory to represent the internal noise of the system. A very similar result is given by equation (2) to that obtained by Cane and Gregory with equation (1). At the absolute threshold, where $I = 0$, taking exponents in equation (2), and ignoring A_2, which is the total visual field, we get:

$$1 + \frac{\Delta I}{k} = e^{(m + b)V^{1/2}A^{-1/2}} \qquad (3)$$

Defining the unit of I so that $k = 1$, and putting $(m + b)V^{\frac{1}{2}} = d$, we get for the absolute threshold:

$$\Delta I = d/A^{1/2} + d^2/2A + d^3/6A^{3/2} \ldots \qquad (4)$$

Cane and Gregory obtain Ricco's law by multiplying throughout by A, and Piper's law by multiplying by $A^{1/2}$. In each case a constant appears among the terms on the right. However, it has been pointed out to me by Dr. Marriott (private communication) that in the first case it is the second term (as given in equation (4)) which provides the constant and in the second case the first term. This causes the following difficulty: the values of the terms on the right depend on the values of both d and A. While d and A can have values such that the first term is considerably larger than any other term, and decrease in the value of A will increase the size of the second term relative to the first term, it will cause the values of other terms to increase as well, and at no time will the second term be more than a fraction of the total value of ΔI (for example, if $d = 1$, $A = 10$, the first terms are 0·3, 0·05, 0·005. . . . If A is made 0·1, the terms are 3·2, 5·0, 5·3 . . .). Thus the unmodified assumption of a simple logarithmic transformation of the stimulus, so far from leading to Ricco's law, appears to exclude it.

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SOCIOLOGY

Conflicting Hypotheses on the Growth of World Population

Over the past twenty thousand years, the logarithm of human population of small areas such as England and Wales—when plotted against time—shows two or more special upsurges, or kinks; for example, due to the introduction of agriculture, or since A.D. 1500. One may recognize three main demographic stages[1]. With the same idea, Prof. E. S. Deevey of Yale proposes a striking plot of total world population, featuring two such kinks in a very pronounced manner[2]. But the "overall geographical speed of cultural diffusion", in early times, was slow; often between 0·5 and 5 km. per annum[3]. Consequently different small areas had their main kinks at widely different times; and they will practically disappear on summation. Hence, with due allowance for the different scales used, the relatively smooth curve for world population[4] seems likely to be much nearer the truth than Deevey's of the following September.

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