of the crystals since the crystals would have a surface area in the region of 50 m.^{2}/gm.

It appears that about 75 per cent saturation with lithium and heating to 190° C. are able to prevent crystalline expansion. At 56 per cent lithium saturation there is some penetration into intracrystalline spaces but large crystalline swelling is not observed. At 25 per cent lithium saturation, montmorillonite exhibits large physical swelling as the result of expansion beyond a potential barrier at a silicate sheet separation of 19 Å.

It can be concluded that large crystalline swelling is not obtained at a surface density of charge of about $0{\cdot}6 \times 10^{-7}$ m.equiv./cm.^{2}, but it is obtained at a surface density of charge of 1×10^{-7} m.equiv./cm.^{2}.

We are indebted to Dr. K. Norrish for the determination of strontium.

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MATHEMATICS

Theory of Optimal Games

Theory of games, and also theory of fair games, are well-known subjects, but a theory of good (or optimal) games does not seem to exist.

To theory of optimal games there would, for example, belong the question: Why, among serious poker-players, is it accepted as 'better' to play the game with a full pack and a hand of five cards rather than with, say, two Piquet packs shuffled together and a hand of seven cards ? (The historical reason for this acceptance is not known, for the origin of poker is as obscure as is that of its name.)

That a game may possess different states of goodness is well seen from the varieties of two-handed whist. These may be epitomized thus: n full packs are shuffled together and equally divided among the two players; of the $26n$ cards that each player receives, kp are dealt face down to form k packets of p cards each; k are dealt face up, one on top of each packet, and the remaining $26n - k(p + 1)$ cards form the player's hand, from which, before the trump is declared, he must throw away r cards face down. When one of the k top cards is played, the card underneath it is turned up; and so on. In these games, obvious examples of zero goodness are (for all n), first, $k = r = 0$, and, secondly, $p = r = 0$, $k = 26n$; whereas, empirically, $n = 2$, $p = 3$, $k = r = 8$ 'makes rather a good game'.

In the context of theory of optimal games, it may be that croquet should receive consideration, for, in its (fairly short) history, it has, more than once, been felt non-optimal, and its controllers have altered the rules fundamentally (the most traumatic alterations being, perhaps, the decisions, first, to abandon play by rotation of colour, secondly, to award an optional lift against a player who is 'doing too well').

There would arise also the question: How far is competition a prerequisite of goodness ? In patience games, this element is, naturally, lacking, yet many of the games are clearly 'very good', most notably, perhaps, those of the *Miss Milligan* series (of which Miss Milligan itself is, by trial and error, the optimal member). Croquet is a competitive game: the player of black-and-blue achieves success if he 'goes round' before the player of red-and-yellow; but there seems no reason why a 'croquet-patience', in which the two players combine to get all four balls round in as few turns as possible, should not be an equally good game.

Clearly, the great difficulty in theory of optimal games will be the establishment of a criterion, or measure, of goodness, and it may be that this establishment will not fall within the domain of mathematics; but, once a measure has been established, some mathematical analysis or, at least, a mathematical envisagement, of the problem might well be possible.

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A Remark on Stochastic Path Integrals

Various stochastic functionals of the type:

$$S(t) = \int_0^t V(X(u), u)\, du \qquad (1)$$

where $X(u)$ is a stochastic process, have been studied in recent years (see, for example, Kac[1]). It seems worth noting that for $X(u)$ Markovian, there is often no particular difficulty in setting up characteristic function equations, of the type derived in my book[2] on stochastic processes (see especially the end of chapter 3), applicable not only to (1) but also to rather more general types of integral. Thus for:

$$S(t) = \lim_{\Delta u \to 0} \sum_{u=0}^{t} v(X(u), u, \Delta X(u), \Delta u) \qquad (2)$$

provided

$$E\,[\{\exp(i\theta\Delta S + i\varphi\Delta X) - 1\} \mid X(t) = x] = \Psi_S\,(i\varphi, i\theta, t, x)\Delta t + o(\Delta t)$$

we may set up an operator equation of the form:

$$\frac{\partial C}{\partial t} = \Psi_S\,(i\varphi, i\theta, t, \frac{\partial}{\partial i\varphi})\, C \qquad (3)$$

where $C \equiv E\,\{\exp(i\theta S(t) + i\varphi X(t))\}$. A particular case given on page 86 of my book is $v \equiv X(u)\Delta u$. As further examples, consider $v \equiv [\Delta X(u)]^2$, for (i) the normal linear Markov process (which includes 'Brownian motion' as a special case) and (ii) the simple birth-and-death process, for which $S(t)$ records the total number of transitions. For (i), $S(t)$ then increases regularly with t (as is well known); for (ii), C is readily evaluated, either by (3) or even more easily from the corresponding 'backward' equation. A further amplification of these points will be given elsewhere.

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