LETTERS TO THE EDITORS

ASTRONOMY

Roughness of the Moon as a Radar Reflector

In a recent communication[1] Hughes has suggested that the law of scattering of radio waves by the lunar surface is consistent with scattering by a rough surface which has irregularities much greater than the radio wave-length, and such that the root mean square value of the slope is about 1 in 20, or 3°. The law of scattering as a function of angle of incidence which, on this model, would be the same as the distribution of the slope of the irregular surface, was found from radar observations to be given by:

$$\sigma = \sigma_0 \exp\{-10\theta\}$$

where σ is the effective scattering area and θ the angle of incidence.

It is of interest to compare this with the distribution of the slope of the lunar surface as determined optically. Fujinami, Ina and Kawai[2] have made photometric measurements of the profile of the Moon's silhouette, as observed during a partial eclipse. Measurements of the elevation of the lunar surface in seconds of arc were made for every degree around the limb. It is a simple matter to obtain a value for the slope of the lunar surface from each successive pair of readings. The distribution of these values, without regard to sign, is shown as a histogram in Fig. 1. The exponential law $\exp(-10\theta)$ is shown as a dotted line for comparison. It will be seen that there is close agreement except for angles less than 1°. (The law proposed by Hughes was stated to apply over the range 3–14°.) The root mean square value of the slope, as determined from the optical measurements, is 3·3°.

This agreement is interesting; but it is, perhaps, surprising, because the optical measurements refer only to structure with a horizontal scale larger than about 30 km. The lunar surface could contain structure of a smaller scale, which would not be resolved optically, but which was larger than the radio wave-length. If the root mean square value of the slope of this smaller-scale structure were comparable with or larger than 3°, the overall distribution of the slope would extend to angles too large to fit the radio observations. The fact that agreement is obtained when only structure larger than 30 km. is considered suggests that the smaller structure, if it exists, has a root mean square value of slope considerably less than 3°.

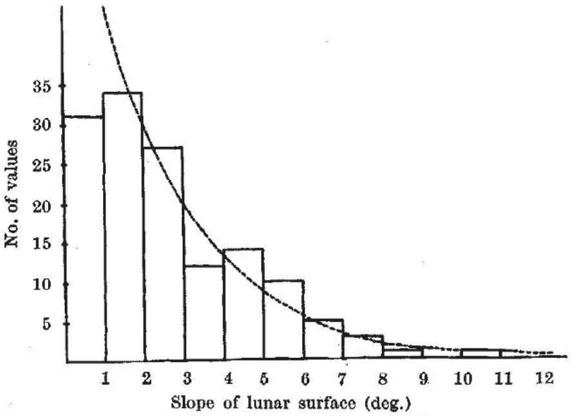

Fig. 1. The distribution of the values of the slope of the lunar surface as determined from the optical measurements of Fujinami, Ina and Kawai. The dotted line is the exponential function $\exp(-10\theta)$

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[1] Hughes, V. A., *Nature*, **186**, 873 (1960).
[2] Fujinami, S., Ina, T., and Kawai, S., *Pub. Astro. Soc. Japan*, **6**, 67 (1954).

GEOPHYSICS

Evaluation of the Second, Fourth and Sixth Harmonics in the Earth's Gravitational Potential

In an article[1] in *Nature* in 1958, it was explained how the rate of rotation of the orbital plane of an Earth satellite (the rate of regression of the nodes) could be used to evaluate the even harmonics in the Earth's gravitational potential U. At an exterior point distant r from the Earth's centre and having co-latitude θ, U may be expressed in terms of Legendre polynomials P_n as:

$$U = \frac{GM}{r}\left\{1 - \sum_{n=2}^{\infty} J_n \left(\frac{R}{r}\right)^n P_n(\cos\theta)\right\} \quad (1)$$

where G is the gravitational constant, M the mass of the Earth, R its equatorial radius, and the J_n are constants to be determined. J_1 is zero if the equator is chosen to pass through the Earth's centre of mass. The motion of a satellite in the gravitational field specified by equation (1) has been investigated theoretically[2–4], and the rate of rotation of the orbital plane, $\dot{\Omega}$, may in principle be expressed in the form:

$$\dot{\Omega} = \sum_{n=2}^{\infty} J_n F_n + \sum_{m=2}^{\infty}\sum_{n=2}^{\infty} J_m J_n F_{mn} + \ldots \quad (2)$$

where the F_n and F_{mn} are functions of the orbital elements. Since J_2 is of order 10^{-3}, the J_n are of order 10^{-6} when $n > 2$, and the F_n are of the same order as the F_{mn}, only the $J_2{}^2$ term in the second series in equation (2) need be considered, and even in that term an approximate value of J_2 is adequate. Thus each observed value of $\dot{\Omega}$ provides one linear relation between the J_n, and observed values from k satellites provide k simultaneous equations between the J_n. These equations are not ill-conditioned if the k orbits differ widely enough. It happens that the odd-numbered J_n have little effect on $\dot{\Omega}$, and the available values[5] for J_3 and J_5, namely:

$$\left.\begin{aligned} J_3 &= (-2\cdot4 \pm 0\cdot3) \times 10^{-6} \\ J_5 &= (-0\cdot1 \pm 0\cdot1) \times 10^{-6} \end{aligned}\right\} \quad (3)$$