major importance when the array is used both for transmission and reception. Under such conditions the effective beam-width is $8^{\circ}$ (to half-power points), and the lobe at $-55^{\circ}$ is 19 decibels down with respect to the main beam, thus giving a considerable increase in azimuthal resolution over the use of a single Yagi. The back lobe is 24 decibels down with respect to the forward beam.

The system is rotated by two 1 horse-power electric motors hydraulically coupled and geared down to drive one of the wheels on each outer bogie. Normal speed of rotation is one revolution in 3 min .

Fig. 3 shows an example of a range-azimuth display obtained using the array, which illustrates the particular advantage of the narrow beam for the purpose of identifying aspect-sensitive reflexions.

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## Radar Determination of the Scattering Properties of the Moon

Information about the scattering properties of the Moon's surface has been deduced by Evans ${ }^{1}$ from the rapid fading of lunar radio echoes. His results were obtained by picturing the surface as an assembly of randomly spaced scatterers. Such a concept, however, does not take into consideration the distribution in depth of the scattering elements. An approach which does so is that adopted by Feinstein ${ }^{2}$, who used Huygens's principle to study the general problem of reflexion from a plane surface having random irregularities that are a function both of the space co-ordinates and of time. With minor modifications to take into account the curvature of the reflecting surface, Feinstein's results may be applied to lunar radio echoes.

Consider first the lunar surface to be corrugated in one dimension only, so that its height $h$ above a mean level can be represented by the random variable $h(x, t)$, where $x$ is distance measured along the surface from its intersection with the line connecting the centres of the Earth and the Moon. If the resultant surface velocity due to the lunar librations is $v$, and if the co-ordinate plane is determined by this vector and the line of centres, the function $h$ will have the form $h(x-v t)$.

Now assume that $h$ is normally distributed and that the joint distribution of the $h$ 's at any two points is bivariate normal. The space correlation function of the signal envelope observed by spaced receivers on the Earth's surface when the Moon is directly overhead can then be shown to be :

$$
\begin{equation*}
C(s)=\exp \{-a[1-\rho(s R / 2 D)]\} \tag{1}
\end{equation*}
$$

where $s$ is separation between receivers, $a$ is $4 k^{2} h^{2}$, $k$ is $2 \pi / \lambda$, where $\lambda$ is the radar wave-length, $\vec{h}^{2}$ is variance of the distribution of $h, \rho$ is normalized space correlation function of the Iunar surface, $R$ is radius of the Moon, $D$ is Earth-Moon distance.

By fitting experimental data to (1), it is possible to determine both the lunar space correlation function $\rho$ and the constant $a$, since $C(s)$ approaches $\exp (-a)$ asymptotically. It is important to note, however, the occurrence of an approximately 500 -fold change in the scale of this function that results from the reflexion of the incident wave-front at the curved lunar surface. Space diversity tests carried out by the U.S. Army Signal Research and Development Laboratory over its circuit from Belmar, N.J., to the University of Illinois, at Urbana, Ill., have verified this expansion in scale.

The autocorrelation function of the received signal envelope at one location on the Earth's surface is given by :

$$
\begin{equation*}
P(\tau)=\exp \{-a[1-\psi(\tau)]\} \tag{2}
\end{equation*}
$$

where $\psi(\tau)$ is the autocorrelation function that would be observed at any point on the mean lunar surface that is fixed relative to the co-ordinate system postulated. Because of the relation between $x$ and $t$, it is immaterial whether we determine $P(\tau)$ or $C(s)$ experimentally, since both contain the same information. Only $C(s)$, however, is expanded by the factor $2 D / R$.

The one-dimensional results for the correlation functions are directly applicable to the two-dimensional case, if the variance of $h$ is independent of direction. For the determination of the distribution of angular power, however, the lunar surface must be treated as two-dimensional, and, in addition, the form of the function $\rho$ must be specified. Feinstein ${ }^{2}$ has derived the angular distribution for the case where:

$$
\begin{equation*}
\rho=\exp \left(-d^{2} / l^{2}\right) \tag{3}
\end{equation*}
$$

where $d$ is distance measured in any direction along the lunar surface and $l$ (or more commonly $l / \sqrt{ } 2$ ) is called the structure size. Because surface irregularities much smaller than $\lambda$ will not appear in the reflected wave-front, the smallest value of $l$ determinable from radar observations will be of the order of magnitude of $\lambda$. If Feinstein's results are again modified to fit the lunar problem, the angular power distribution is found to be :

$$
\begin{equation*}
p(r)=(\exp -a)\left[\cos \left(k r^{2} / R\right)+\sum_{n=1}^{\infty} \frac{a^{n}}{n \cdot n!} \exp \left(-r^{2} k^{2} l^{2} / n R^{2}\right)\right] \tag{4}
\end{equation*}
$$

In order to determine $p(r)$ completely, we must first assume that $\rho(d)$ has the form given in (3) and obtain numerical values of $a$ and $l$ by fitting observational data to (1) or (2). These values of $a$ and $l$ are then substituted into (4).

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