$$\alpha_i^S = (\delta_{ij} - W_{ij})\,\alpha^j = A_{ij}\alpha^j \text{ say} \quad (1)$$

W_{ij}, α_i^S may be positive or negative according as their synaptic endings are excitatory or inhibitory.

Consider n outputs of receptors from a sensory field S such that the covariance :

$$C_{ij} = \lim_{T\to\infty} C_{ij}(T) = \lim_{T\to\infty} \frac{1}{T}\int_0^T \alpha_i^S(t)\,\alpha_j^S(t)\,dt \quad (2)$$

exists. The structure of this matrix reflects the simplest properties of the field S, namely, if the rank $r<n$ then there will be $n-r$ linear relations between the α_i^S. If now we can arrange that :

$$\lim_{t\to\infty} \sum_k A_{ik}(t)\,A_{jk}(t) = C_{ij} \quad (3)$$

then we shall have a description of the field $S(t)$ in terms of n outputs $\alpha^j(t)$ which are orthonormal in the sense that :

$$\lim_{T\to\infty} \frac{1}{T}\int_0^T \alpha_i(t)\,\alpha_j(t)\,dt = \delta_{ij} \quad (4)$$

and of which therefore $n-r$ must be arbitrary, that is, generated by the net itself. Were it not for this, which will allow us to distinguish them from the r outputs representing $S(t)$, we should have gained nothing from the transformation (1).

To realize (3), let us put :

$$t\frac{\partial A}{\partial t}ik + \frac{1}{2}A_{ik} = \frac{1}{2}\alpha_i^S\alpha_k \quad (5)$$

Multiplying through by A_{jk}, using the symmetry of C_{ij} and summing over k it is easy to show that this satisfies (2) and (3). However, (3) only determines A_{ij} up to an orthogonal transformation corresponding to an arbitrary rotation of the orthogonal frame ; the additional condition:

$$_k\Sigma\, A_{ki}A_{kj} = \lambda_i\,\delta_{ij} \quad (6)$$

determines the principal axis solution, where λ_i are the proper values of C_{ij}. If (6) holds, then we have :

$$A_{ij}\,\alpha_i^S = \lambda_j\,\alpha_j \quad (7)$$

multiplying (5) by A_{ij} and summing this time over i, one can check that (6) follows.

Using (1) and (4), the corresponding condition for the connexions is :

$$t\frac{\partial W}{\partial t}ij + \frac{1}{2}W_{ij} = \frac{1}{2}(\alpha_i - \alpha_i^S)\alpha_j \quad (8)$$

which means that the (negative) connexion between two cells is proportional to the correlation between the activity of the cell from which the connexion is coming and the stimulus on the cell to which it is going. For the feedback of a cell to itself the same is true of the reciprocal gain. This shows that if α_j is uncorrelated with any of the α_i^S, then the cell j is effectively isolated and its gain rises to an infinite value such that α_j is a self-generated signal of unit variance. Under these conditions an indefinitely flat peak in the frequency response will produce a single-frequency sinusoidal output. If no two of these frequencies are exactly equal, then (4) will certainly be satisfied. It is not unreasonable to compare this behaviour with the α-rhythm of the inattentive brain. Furthermore, the principal axes solution in which the co-ordinate axes are determined by successive extrema of the stimulus vectors is the one most indicated by physiological evidence.

An example of the application of such a net would be the following : suppose we have n audio receptors responding to a frequency band $\Delta\nu_i$, and let their (rectified and smoothed) outputs be $\alpha_i^S(t)$. For a sensory field such as normal speech, the latter will be highly correlated, so that a smaller number r will be able to convey the information. In fact, just such a procedure has been used[7] in a speech communication channel known as the 'Vocoder'. For example, with $\Delta\nu_i = 15$ c./s., $n = 16$, we can have $r = 10$ without loss of fidelity.

However, the significance of the net as a model of local brain activity would appear to be the following. If we drop the rigour implied in the infinite limit of (2), then what the net does for us is to factorize a sensory field into a part which has no constancy in time and a part which is invariant over a period of time T, and which is expressed in the values of the A_{ij}. Over longer periods of time, the latter may vary and could be the subject of analysis by a net of longer time constant. In this way knowledge is possible in spite of the conditional and incomplete nature of all relations.

Note added in proof. To obtain the principal axis solution uniquely some modification of (5) is required ; it will be given elsewhere.

M. C. GOODALL

Cornell Computing Center,
Ithaca, New York.

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Standardized t

STUDENTS' t may be standardized by expressing it in units of its own standard deviation; thus, $t_s = t\sqrt{\{(\nu-2)/\nu\}}$, where ν is the number of degrees of freedom. The procedure reduces the variability of the tails with ν for a given probability, P, and in particular, for $P = 0 \cdot 05$ and $\infty \geqslant \nu \geqslant 4$, t_s lies between $1 \cdot 96$ and $2 \cdot 0$ whereas t lies between $1 \cdot 96$ and $2 \cdot 776$. This finding extends the applicability of certain 'large' sample methods down to 4 degrees of freedom. The standardizing is extremely simple to apply—merely divide the error sum of squares by $\nu - 2$ instead of by ν and use this value to calculate the appropriate standard error (SE). For example, if d is the difference between two means and $d \geqslant 2\ SE$, $P \leqslant 0 \cdot 05$, that is, the difference is significant at the 5 per cent level. If $d < 1 \cdot 96\ SE$, $P > 0 \cdot 05$ and the difference is not significant. Between these two limits of d, $P \simeq 0 \cdot 05$; actually $0 \cdot 0535 > P > 0 \cdot 0455$. Again, $\pm 2\,SE$ will give a close approximation to 95 per cent confidence limits (actual value between 95 and $95 \cdot 45$ per cent). When the formula is applied to the correlation coefficient, r, we find that r is significant at the 5 per cent point if $r \geqslant 2/\sqrt{(\nu+2)}$.

JOHN B. DE V. WEIR

The University,
Glasgow, W.2.