Influence of the Thomson Effect on the θ–φ Relationship for a Constrictive Resistance in Thermal Equilibrium

THE relationship between the temperature θ and the electrical potential φ at any point within a current-carrying constrictive resistance in thermal equilibrium can be expressed in the form[1]:

$$\int_{\theta}^{\theta_1} \frac{\lambda}{\chi}\,d\theta = \tfrac{1}{2}\varphi^2 - \int_{0}^{\varphi}\int_{\theta}^{\theta_1} \sigma\,d\theta\,d\varphi \qquad (1)$$

provided that the material is both homogeneous and isotropic.

In this equation λ and χ are the thermal and electrical conductivities respectively, and σ is the Thomson coefficient. At the warmest section in the constriction the temperature is θ_1 and the potential φ is arbitrarily taken to be zero.

For most metals the ratio of the thermal and electrical conductivities is roughly proportional to the absolute temperature T, and we have the Wiedmann–Franz–Lorentz equation[2]:

$$\frac{\lambda}{\chi} = AT$$

The Thomson coefficient σ is likewise approximately proportional to the temperature[3], except at temperatures near the melting point, and we can write:

$$\sigma = BT$$
$$= \frac{\tau\lambda}{\chi}$$

where A, B and τ are constants.

The θ-φ relationship can then be re-written in the form:

$$\int_{\theta}^{\theta_1} \frac{\lambda}{\chi}\,d\theta = \tfrac{1}{2}\varphi^2 - \tau\int_{0}^{\varphi} d\varphi \int_{\theta}^{\theta_1} \frac{\lambda}{\chi}\,d\theta$$

If we define an operator Q to be such that $Q\,f(\varphi)$ is

$$\int_{0}^{\varphi} f(\varphi)\,d\varphi$$

then the foregoing equation becomes:

$$[1 + \tau Q]\int_{\theta}^{\theta_1} \frac{\lambda}{\chi}\,d\theta = \tfrac{1}{2}\varphi^2$$

so that:

$$\int_{\theta}^{\theta_1} \frac{\lambda}{\chi}\,d\theta = \frac{1}{[1+\tau Q]}\tfrac{1}{2}\varphi^2$$
$$= \tfrac{1}{2}[1 - \tau Q + \tfrac{1}{2}\tau^2 Q^2 - \tfrac{1}{3}\tau^3 Q^3 + \ldots]\varphi^2$$
$$= \frac{1}{2!}\varphi^2 - \frac{\tau}{3!}\varphi^3 + \frac{\tau^2}{4!}\varphi^4 + \ldots$$
$$= \frac{1}{\tau^2}[\exp(-\tau\varphi) + \tau\varphi - 1] \qquad (2)$$

Thus:

$$\int_{\theta}^{\theta_1} \frac{\lambda}{\chi}\,d\theta = \int_{T_\theta}^{T_{\theta_1}} AT\,dT = \tfrac{1}{2}A[T_{\theta_1}^2 - T_\theta^2]$$
$$= \frac{1}{\tau^2}[\exp(-\tau\varphi) + \tau\varphi - 1] \qquad (3)$$

If the product τφ is small, this result becomes to a close approximation:

$$A[T_{\theta_1}^2 - T_\theta^2] = \varphi^2\left[1 - \frac{\tau\varphi}{3}\right] \qquad (4)$$

Equation (1) corresponds with that part of the constriction in which the electric current flows in the direction of decreasing temperature. In that part where the current flows in the direction of increasing temperature, the algebraic sign of the Thomson coefficient is reversed, and in this part of the constriction the θ-φ relationship is:

$$\tfrac{1}{2}A(T_{\theta_1}^2 - T_{\theta'}^2) = \frac{1}{\tau^2}[\exp(\tau\varphi) - \tau\varphi - 1] \qquad (5)$$

so that: $$A(T_{\theta_1}^2 - T_{\theta'}^2) = \varphi^2\left[1 + \frac{\tau\varphi}{3}\right] \qquad (6)$$

if τφ is small.

W. DAVIES

Department of Engineering,
University College,
Newport Road,
Cardiff.

[1] Jones, F. L., "Fundamental Processes of Electrical Contact Phenomena" (H.M.S.O., 1953)
[2] Sommerfeld, A., and Bethe, H., "Elektronentheorie der Metalle", in "Handbuch der Physik" (Springer, 1933).
[3] Fowler, R. H., "Statistical Mechanics".

Resolution of Wide-range Grating Spectrometers

WHILE it is common experience that the wave-number resolution of grating spectrometers tends to be constant over a wide wave-length range[1], the theoretical basis for this observation does not appear to have been clearly formulated. The reason for this omission is doubtless due to concentration of interest on a particular grating, or set of gratings, and to the diversity of sources of radiation and detectors used for different spectral regions. However, if an unlimited range of gratings be assumed, so that maximum diffracted energy can always be assured, and attention is confined to a black-body source, some simple relationships may readily be deduced.

From the general form of the grating equation

$$d(\sin\theta_1 + \sin\theta_2) = n\lambda \qquad (1)$$

where d is the grating spacing, θ_1 the angle of incidence, θ_2 the angle of diffraction, λ the wave-length and n the order of the spectrum, it is obvious that for a given geometrical configuration λ in the first order is proportional to d. Only the first order need be considered since a grating in the nth order is, for our present purpose, equivalent to a grating with spacing d/n in the first order. The spectral interval obtained with a grating in a given spectrometer is proportional both to d, as can be seen by differentiating equation (1), and to the slit-width, s.