cally any two coins are indistinguishable; but that each has available a number of distinguishability states, and provided they are not both in the same distinguishability state, we can use the properties of these states to tell them apart.

I assume that we are dealing either with Fermi coins or with Bose coins. Let M be the number of distinguishability states available to one coin; that is, let the 'heads' and 'tails' states be M-fold degenerate. We now define the *indistinguishability parameter* μ by $\mu = \pm 1/M$; the upper sign for Fermi coins, the lower for Bose coins. For the two-coin system, there are M^2 possible distinguishability state vectors, of which $1/2\,(M^2 + M)$ are symmetric and $1/2\,(M^2 - M)$ antisymmetric. In terms of the indistinguiability parameter, the number of symmetric states is $1/2\,M^2\,(1 \pm \mu)$ and the number of antisymmetric states is $1/2\,M^2\,(1 \pm \mu)$, where again the upper and lower signs apply respectively to Fermi and Bose coins.

There are, on the other hand, exactly four heads-tails state vectors, namely:

$$\begin{array}{ll} HH & (+) \\ TT & (+) \\ HT + TH & (+)^* \\ HT - TH & (-)^* \end{array}$$

The signs in parentheses indicate the symmetry, and the starred vectors are the ones containing one head and one tail.

Since the total state vector must be symmetric or antisymmetric for Bose or Fermi coins respectively, we find that the total number of possible state vectors is:

$$[3\,M^2\,(1-\mu)/2] + [M^2\,(1 + \mu)/2] = M^2\,(2-\mu);$$

whereas the number of state vectors with one head and one tail is:

$$[M^2\,(1-\mu)/2] + [M^2\,(1 + \mu)/2] = M^2$$

Thus we find that the probability of finding one head and one tail is:

$$p_{HT} = 1/(2-\mu).$$

This is plotted against the indistinguishability parameter in Fig. 1. The circles indicate the possible values of μ, determined by the fact that $1/\mu$ must be a positive or negative integer. The three limiting cases of the first paragraph are apparent.

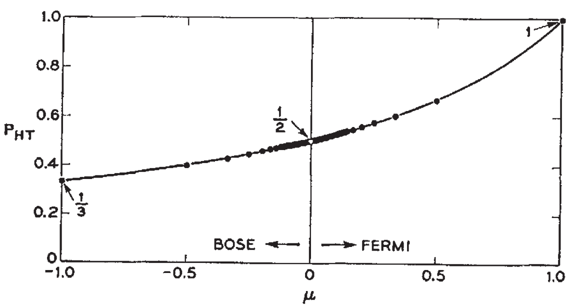

Fig. 1. Probability of tossing one head and one tail as a function of the indistinguishability parameter.

As an illustration, consider the case of two coins which are completely indistinguishable except that each has a spin S; this gives $p_{HT} = (2S + 1)/(4S + 2 \mp 1)$. Thus the probability of getting one head and one tail is 2/3 for Fermi coins of spin one-half and 3/7 for Bose coins of spin one. For very large spins, the probability approaches its classical value.

GABRIEL WEINREICH

Bell Telephone Laboratories,
Murray Hill, New Jersey, U.S.A.

Loss of Information brought about by Loss of Data

Loss of data from an experiment can diminish the accuracy of estimation of a treatment mean in two ways, by reducing (1) the number of observations on which the mean is based, and (2) the efficiency of estimation. It is assumed that the design was initially orthogonal, and that missing plot values are to be substituted in the usual way. Taylor[1] has studied the effect of missing plots on the standard errors of the differences of treatment means in randomized block designs; recently I needed to know the effect on the errors of the means themselves.

To this end I worked out exact solutions in two cases, (1) where plots are missing for all combinations of certain blocks and treatments, the data being otherwise complete, and (2) where no two of the plots missing have a block or treatment in common. The results suggested an approximate solution in the general case, as follows: If a trial is designed with b blocks and t treatments, so that it is expected to have $f = (b-1)(t-1)$ degrees of freedom for error, then, if a plot is missing from treatment p and block q, and there are (1) j plots missing in all from treatment p, (2) k plots missing in all from block q, and (3) h plots missing from treatments other than p but in blocks containing an existing plot of p, the loss of effective replication brought about by the loss of this plot is not one, but:

$$1 + \frac{b-j}{b(t-k)+j} + \frac{hbt}{f(f+t)^2}.$$

The last term is usually negligible.

I have tested this approximation in a number of instances, and it has always given the effective replication correct to within 2 per cent, provided (1) not more than one-fifth of the total data of the experiment are missing, and (2) each block retains at least two plots. (If a block retained only one, it were better omitted anyway.)

To take an example, let there be four blocks each of five treatments, and four missing plots disposed as follows:

Treatment	A	B	C	D	E
Block I	M	M	.	.	.
II	M	.	.	.	.
III	.	M	.	.	.
IV	.	.	.	.	.

According to the approximation the effective degree of replication of treatment B is:

$$4 - \left(1 + \frac{2}{4.3+2} + \frac{4.5}{12.17^2}\right) - \left(1 + \frac{2}{4.4+2} + \frac{4.5}{12.17^2}\right).$$

which equals 1·73. The exact figure is 1·75.

S. C. PEARCE

East Malling Research Station,
Kent.

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[1] Taylor, J., *Nature*, 162, 262 (1948).