LINEAR OPERATIONS AND CLASSICAL ANALYSIS

Linear Operators

By Nelson Dunford and Jacob T. Schwartz. Part 1: General Theory. (Pure and Applied Mathematics: a Series of Texts and Monographs, Vol. 7.) Pp. xiv+858. (New York: Interscience Publishers, Inc.; London: Interscience Publishers, Ltd., 1958.) 165*s*.

THE notion of a linear operator (transformation) is familiar to most mathematicians; but the extremely ramified and elaborate theory of such operators, as presented in the work under review, will come as a surprise to many who have not specialized in this field. Part 1 alone of the two-volume work covers 850 pages, including more than 100 pages of bibliography. However, many different parts of mathematics, particularly hitherto rather isolated branches of classical analysis, are brought together under the strong unifying influence of the theory; and the mathematician who has grown tired of exploring one peak after another may take comfort in coming upon an edifice which acts like an umbrella covering a vast area.

Part 1 presents the topological theory of spaces and operators, and the spectral theory of arbitrary operators (the theory of completely reducible operators is deferred to Part 2). Chapter 1 deals with concepts and results from set theory, topology and algebra, and Chapter 2 with the three basic principles of analysis. These consist of (*a*) the principle of uniform boundedness, which involves the result that the limit of a sequence of continuous linear operations is continuous, (*b*) the interior mapping principle, which asserts that in the case of certain spaces continuous linear mappings map open sets on to open sets, (*c*) the Hahn-Banach theorem, which is concerned with the existence of extensions of a linear functional. Chapter 3 deals with integration and set functions, and Chapter 4 with twenty-eight spaces (mostly Banach) which occur frequently in analysis. These include various sequences and the space of all bounded continuous real or complex functions defined on a normal topological space. The notion of convexity in a general linear space is treated in Chapter 5. It is shown how various classes of linear functionals determine topologies in a linear space, including a weak topology in a Banach space. Special attention is given to the compactness properties and reflexivity of these topologies, and some fixed point theorems are derived. In Chapter 6 linear maps between Banach spaces are studied. The notions of adjoint and projection operators and also weakly compact and compact operators are introduced and the basic properties of the operators are obtained. Representations of classes of operators are given in the spaces of continuous and integrable functions. The concepts and methods of Chapter 7 centre around the idea of the spectrum of an operator. Earlier chapters are concerned with topological aspects of operator theory, but function theoretic and algebraic methods are used in the spectral theory. Perturbation theory covering changes in the spectrum and the resolvent operator is developed, the basic theorem here being due to Rellich. Chapter 8, the last of Part 1, is concerned with applications to semi-groups of operators, functions of an infinitesimal generator, and ergodic theory.

A question one might well ask, following upon the presentation of so much theory is: What does it do? The answer is contained in the large number of graded exercises, which, the authors point out, are not 'drill' problems but carrying on the text. In fact, they present the applications of the theory to familiar branches of classical analysis, including summability of series (Chapter 2) and integrals (Chapter 4), orthogonal expansions, particularly Fourier series (Chapter 5), and inequalities (Chapter 6). It is interesting to note, and indicative of the power of the theory, that convexity methods, depending largely on the Reisz convexity theorem, may be used to derive many of the most familiar and important inequalities of analysis.

"Linear Operators" is encyclopædic in character and will probably become the standard reference text in its field. Parts of it may also be used in courses for both undergraduates and those entering upon research. In a work of this size material tends to become buried; but there is a detailed subject index which will help to prevent this and the "Notes and Remarks" at the end of each chapter form a very good guide to the extensive literature.

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MATHEMATICS AND OPERATIONS RESEARCH

Mathematical Methods of Operations Research

By Thomas L. Saaty. Pp. xi+421. (London: McGraw-Hill Publishing Company, Ltd., 1959.) 77*s*. 6*d*.

OPERATIONS research, which is now generally regarded as a well-defined field of activity moving in its own right, began life in unusual circumstances. The impact of war produced many practical problems, both tactical and strategic, which had to be solved, often within a limited time and with a limited amount of information as data. Men, whose basic training had been in different scientific fields, were put together in teams and their task was to formulate and solve the problems inherent in the practical situations at hand. The result was highly successful, and thus operations research was born. In the post-war years activity has developed greatly on the industrial side. In addition, there has been time for a mathematical penetration of the subject to occur, and the use of mathematics in this field forms the subject-matter of the book under review.

There are four parts, the first being a discussion of a varied collection of topics, including scientific method, logic, and the building of mathematical models. Parts 2 and 3 deal with the more serious mathematical methods of operational research, and cover optimization and programming (both linear and quadratic), the theory of games, probability, statistics and the theory of queueing. These two parts form the core of the book and take up about three-quarters of the volume. They are well written and will give the graduate mathematician a good introduction to operational research problems and the sort of mathematical methods needed in their solution. The chapter on queueing is an excellent summary, and it is a pity that it has not been expanded to deal with the subject more fully. One printing error has been detected: on p. 340 the