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A Non-Thermal Direct-Current Plasma-heating Mechanism

THE power required to heat a plasma to thermonuclear temperature may be greatly reduced if one can apply the ion relaxation principle suggested by Schlüter[1] and also (in a different form) by Motz (unpublished work). In the Schlüter scheme, an oscillation in a magnetic field-strength at a frequency much lower than the frequency of electron collision heats the ions quickly, without appreciably heating the electrons. Since, in a plasma at equilibrium, the electrons are responsible for nearly all the loss of heat, this is a great advantage. In this communication it is shown that, under the influence of an axial unidirectional electric field (as in Zeta and Sceptre), the orbits of electrons in a plasma are distorted in such a way that they feed power to travelling compression waves in the plasma. It is conceivable that, by this means, and with a suitable choice of parameters, a substantial fraction of the total power available to a ring discharge might be converted to ion energy.

Let us suppose that the magnetic field, B, is:

$$B_z = B_0 [1 + \alpha^2(z - v_a t)^2] \qquad (1a)$$

$$B_r = -\alpha^2 B_0 (z - v_a t) r \qquad (1b)$$

with B_0 and α constants, z, r and t position and time co-ordinates, and v_a the velocity of the potential well. (In a gas discharge, v_a will be approximately the Alfvén velocity, $cB_0(\varepsilon_0/\rho)^{1/2}$; see also ref. 2.) An electron moving in this magnetic field can be characterized by its magnetic moment μ, where:

$$\mu = mv_\perp^2/2B \qquad (2)$$

with $v_\perp$ being the component of the velocity normal to B. The parameter μ is very nearly a constant of the motion[2]. We consider motions in the vicinity of the $z - v_a t$, r origin. The restoring force on the electron is:

$$f = -\mu\nabla_{z'} B = -2\mu\alpha^2 B_0 z' \qquad (3a)$$

where $z' = z - v_a t$. The electron thus has simple harmonic motion about $z' = 0$, at an angular frequency ω, where:

$$\omega = (2\mu\alpha^2 B_0/m)^{1/2} = \alpha v_\perp (z' = 0) \qquad (3b)$$

In this motion, the magnitude, v', of the velocity of the electron, measured in the primed frame, is also nearly constant. If a small electric field, $-E_z$, is applied, so that $|eE_z/\alpha m v'^2| \ll 1$, the motion is still nearly harmonic, but the oscillation will now be centred about $z' \simeq eE_z/2\mu B_0 \alpha^2 = z_0$.

For a simple example, suppose the electron orbits in a plane normal to the z-axis at z_0. The force eE_z on the electron is balanced by the magnetic force $ev_\perp B_r$, and in a time-interval Δt the work done on the magnetic field is $\Delta W_z = f_z \Delta z = ev_\perp B_r \Delta z = eE_z v_a \Delta t$. In this case, all the work done upon the electron by the E_z field is passed on to the B field.

In the above example, the radial component of the force did no work. This is also true for the case of an electron which oscillates in z', when we average over a complete cycle. Choosing an appropriate z-axis, we have $mv_\perp = B_z er$, and:

$$\frac{dr}{dt} = \frac{d}{dt}\left(\frac{mv_\perp}{eB_z}\right) = \frac{m}{e}\left[\frac{1}{B_z}\frac{dv_\perp}{dt} - \frac{v_\perp}{B_z^2}\frac{dB_z}{dt}\right] \qquad (4)$$

while, using (2), we have:

$$mv_\perp \frac{dv_\perp}{dt} = \frac{\mu dB_z}{dt} \qquad (5)$$

setting μ constant. Combining (2), (4) and (5), we get $\Delta W_r = \int f_r v_r dt = \int ev_\perp B_z \frac{dr}{dt} dt = \mu \Delta B$, which vanishes, when taken over a complete cycle.

For the work done against B_r in the vibratory case, we use (1b), (2), and the centrifugal force equation, getting $f_z = ev_\perp B_r = 2\alpha^2\mu B_0 z'$. Setting $z' = z_0 + z_m \sin\omega t$, and $v_z = v_a + \omega z_m \cos\omega t$, we have:

$$W_z = \int f_z v_z dt = 2\alpha^2 B_0 \mu \int (z_0 + z_m \sin\omega t) \times (v_a + \omega z_m \cos\omega t) dt \qquad (6)$$

which, after integrating, and substituting for z_0, gives:

$$\Delta W_z = 2\alpha^2 B_0 \mu z_0 v_a \Delta t = eE_z v_a \Delta t \qquad (7)$$

for the work done during a complete cycle.

If E_z is reversed, this effect gives a decrease in the energy of the magnetic perturbation.

It is tempting to generalize this result into the following rough rule: in a plasma, an electron that moves undisturbed through a complete vibratory cycle in its B potential well will divert all the energy it has gained from E_z in the interval to the magnetic field.

The fraction of the electrons in a discharge which will feed energy in this way to the compression waves in B will depend upon the electron temperature and the amplitude oscillations of intensity B. Under suitable conditions, the oscillations will be built up, and, if their frequency is appropriate, they will heat the plasma ions by the Schlüter mechanism. It is possible that such oscillations contribute to the remarkably fast ion heating in current experiments[3].

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