Propagation of Radio Waves from Cosmical Sources

WHEN a radio wave passes through an un-ionized medium its ray path is rectilinear. On passing through such ionized layers, which usually surround the Earth, its trajectory is satisfactorily determined by Snell's law:

$$\mu\rho \sin i = \rho_{\|} = \sin i_0 = \text{constant} \qquad (1)$$

which is valid for any point of the ray path. For the sake of simplicity the radius of curvature ρ_m of the maximum of electron density is taken as the unit of length. The significance of the parameters used follows from Fig. 1.

According to (1) and Fig. 1, the rectilinear part of the ray path must be tangential to the sphere defined by $\rho_{\|}$, everywhere in the un-ionized medium. Thus a ray which passes through spherical ionized layers of the same centre of curvature O is turned about this centre of curvature by the angle R (Fig. 1).

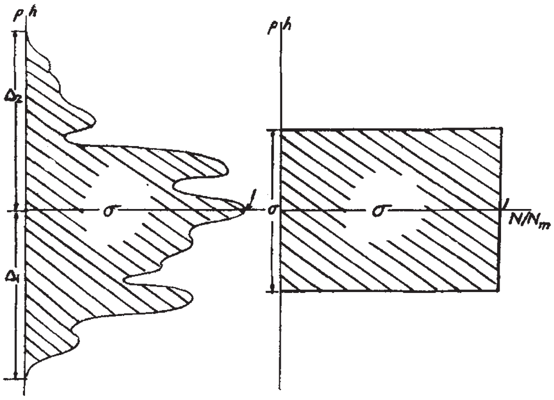

Fig. 2. The true and the equivalent layer, introduced in order to avoid discrepancies that could appear as the electron density is usually expressed in cm.[3] while the introduced unit of length is ρ_m. In this equivalent layer having the same contents of electrons and the same N_m, the electrons are squeezed together in order to form a layer of uniform electron density (N_m). The thickness of this equivalent layer $\sigma > 1$ is to be introduced into formula 2

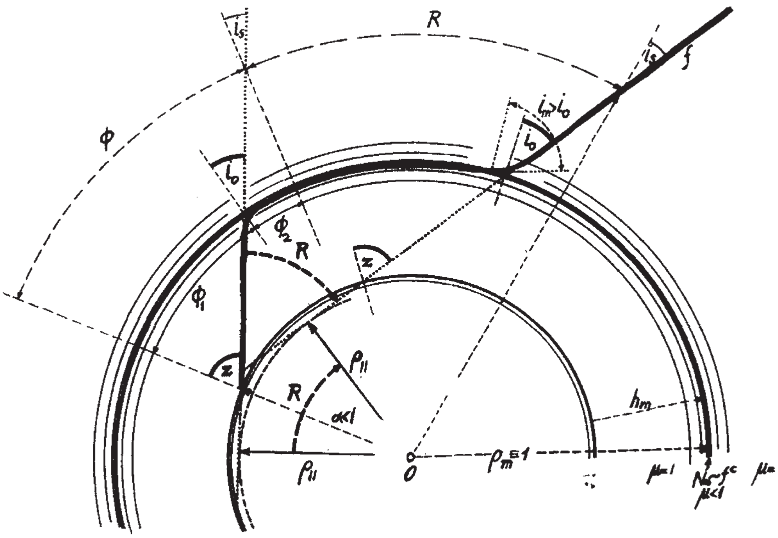

Fig. 1. Parameters used in formulæ 1 and 2. f is the wave frequency, f^c the critical frequency of the layer,

$$\mu = \sqrt{1 - \left(\frac{f^c}{f}\right)^2} \text{ and } \rho = \frac{6{,}400 \text{ km.} + h}{6{,}400 \text{ km.} + h_m}$$, h and h_m being in km.

For this angle of refraction R the following simple formula has been derived[1]:

$$R = \frac{90°}{\pi}\left(\frac{f^c}{f}\right)^2 \frac{\sin i_0}{\cos^3 i_0} \cdot \sigma \qquad (2)$$

where again $\rho_m = 1$, i_0 is the angle of incidence of the rectilinear parts of the ray path, related, however, to this level $\rho_m = 1$, and the equivalent thickness σ of the layer is to be determined according to Fig. 2.

Formula 2 is valid for $\frac{f^c}{f \cos i_0} < 0 \cdot 4$ and $\Delta < \rho_m/5$.

For $f^c/f \cos i_0 > 0 \cdot 4$ the true refraction is greater than the calculated one, and it increases to infinity; this case, dealing with a perpetual propagation of radio waves around the Earth along one of the two different levels of the ionosphere, has been treated previously[2].

The refraction formula 2 has the advantage that it depends only upon the total content of all free electrons present in the layer, but not upon the irregularity in electron distribution with height, which up to the present time has made the calculation of refraction uncertain. Formula 2 is valid even for sharp gradients of electron density, and for parabolic types of layer, $\sigma = \frac{4}{3}\Delta$, it goes over into Bailey's formula[3].

The theoretical result, mentioned above, is in good agreement with experimental results obtained from the signal records of the first artificial satellite. The angular distance of the satellite $\varphi + R$ was determined from formulæ 1 and 2 and Fig. 1 as follows:

$$a \sin z = (a + h_m) \sin i_0$$

$$\sin i_{\text{sat.}} = \frac{a}{a + h_{\text{sat.}}} \sin z$$

$$\varphi_1 = z - i_0, \quad \varphi_2 = i_0 - i_{\text{sat.}},$$

$$\varphi = \varphi_1 + \varphi_2 = z - i_{\text{sat.}}$$

and $\sin i_0 \leqslant \mu_m \rho_m$ corresponds to the minimum of the $\mu\rho$-curve. In accordance with the calculated values the received signal disappeared usually sooner on 20 Mc./s. than on 40 Mc./s., and only in cases when the signal was coming from countries where for 20 Mc./s. $f \doteq f^c$ was long-distance propagation of the signal around the Earth from the magnetic equator observed.

Summarizing, the rectilinear part of a ray path becomes rotated about the centre of curvature of the ionized layer through which it has penetrated by the angle of refraction R. This deviation R depends only upon i_0, f^c/f and the content of free electrons present in the whole layer, but it is independent of their distribution.

E. CHVOJKOVÁ
Astronomical Institute of Czechoslovakia,
Ondřejov. Nov. 9.

[1] Chvojková, E., *Bull. Astron. Czech.*, [9, 1 (1958)].
[2] Chvojková, E., *Bull. Astron. Czech.*, 5, 104, 110 (1954).
[3] Bailey, D. K., *J. Mag. Atm. Elect.*, 53, 41 (1948).