Effect of the Earth's Equatorial Bulge on the Life-time of Artificial Satellites and its use in determining Atmospheric Scale-Heights

PREVIOUS correspondents[1-3] have discussed the derivation of a simple formula for the life-time T of an Earth satellite. The formula concerned is:

$$T = \frac{3}{4}Pe\left(1 + \frac{7}{6}e + \frac{1}{2}kae\right)\Big/\left(-\frac{dP}{dt}\right) \qquad (1)$$

where P is the period, dP/dt the rate of change of period with time, e the eccentricity, a the semi-major axis and $1/k$ the atmospheric scale-height. This formula is valid provided e is both sufficiently small and sufficiently large, and the last two terms on the right-hand side (given here for the first time) indicate the limitations on e.

From various sets of available orbit data, I find that the descent dates predicted by (1) vary with the epoch of the data, more recent data leading to earlier descent dates. For the Sputnik 1 sphere ($1957\alpha_2$), data of October 15 give January 30 ($\pm$ 4), whereas data of November 9 give January 18 ($\pm$ 2). The actual descent date was about January 2. For Sputnik 2 (1957β), data of November 9 give June 24 ($\pm$ 10), whereas data of February 4 give April 30 ($\pm$ 1). Although these dates refer strictly to the time when the orbit becomes nearly circular, they should be only a few days short of the descent dates.

This shortening of the life-time is attributed to the fact that the perigee points of both satellites have been rotating slowly towards the equator and hence, on account of the equatorial bulge, into denser air. The perigee of sphere $1957\alpha_2$ moved from 42° N. to 5° N. during its life-time; whereas for 1957β, the perigee was initially at about 53° N., and will reach the equator by March 19. An equatorial radius is in excess of a radius at latitude 45° by 11 km., and so the bulge is equivalent to an air-density change of about 60 per cent in the 200-km. altitude region. Formula 1 should therefore not be applied without considering the latitude variation of perigee. The neglect of this effect in Scott's[2] estimates happened to compensate for the omission of the last two terms in (1), so leading to surprisingly good predictions.

My attention was first directed to the effect of the equatorial bulge on perigee air-density ρ by the values in Table 1, which were derived from the formula:

$$\rho = \frac{1}{3\varkappa}\frac{(1-e^2)^{1/2}}{(1+e)^2}\left(\frac{2ke}{\pi a}\right)^{1/2}\left(-\frac{dP}{dt}\right) \qquad (2)$$

where $\varkappa = SC_D/m$, S being the effective cross-sectional area and m the mass of the satellite, and C_D the drag coefficient. (This formula is valid so long at e is not too small; and, in all cases considered, the contributions from higher-order forms and the effect of the Earth's oblateness amount to only of the order of a per cent.)

Perigee air-density values cannot be guaranteed to better than about 30 per cent on account of uncertainties in k and $\varkappa$. Density ratios relating to the same satellite object are, however, independent of k and $\varkappa$ and should be accurate to a few per cent. Thus the perigee air-density for 1957β increased between November 9 and February 3 by a factor of $1\cdot51(\pm 0\cdot06)$. The change in perigee distance between two epochs can be shown to correspond to a change of density of approximately

$$\rho/\rho_0 = (ae_0/a_0e)^{1/2} \qquad (3)$$

and this amounts to only $1\cdot17$ ($\pm$ 0·01) for 1957β between November 9 and February 3. The discrepancy between these two factors, amounting to $1\cdot29$ ($\pm$ 0·05), can be satisfactorily accounted for by the increase of 9·8 km. in the Earth's radius between 49° N. and 19° N. The value obtained for the atmospheric scale-height is then 39 ($\pm$ 6) km. The data for perigee-height for 1957β in Table 1 enables the scale-height to be determined as 34 ($\pm$ 11) km.

Table 1. VALUES OF AIR DENSITY IN THE REGION OF 200-KM. ALTITUDE

Object	Sphere, $1957\alpha_2$*	Sphere, $1957\alpha_2$*	$1957\alpha_1$†	Rocket, 1957β‡	1957β‡
Epoch of data	October 15	November 9 (approx.)		February 3	
Perigree latitude	37° N.	26° N.	26° N.	49° N.	19° N
Perigree height (km.)	206 ($\pm$7)	201 ($\pm$4)	202 ($\pm$4)	211 ($\pm$4)	197 ($\pm$1)
Perigree air density (gm./c.c.)§	$5\cdot4\times10^{-13}$	$6\cdot7\times10^{-13}$	$7\cdot3\times10^{-13}$	$4\cdot6\times10^{-13}$	$7\cdot0\times10^{-13}$

* C_D taken equal to 2.

† $\varkappa$ taken equal to 1·8 times that of sphere $1957\alpha_2$ (as derived from the ratio of their initial rates of change of period with time).

‡ $\varkappa$ taken equal to 5–6 times that of rocket $1957\alpha_1$ on account of its greater mass.

§ Scale-height $1/k$ taken to be 25 km. Equation 2 shows that ρ varies inversely as the square root of the scale-height.

For sphere $1957\alpha_2$ the height data above are not sufficiently accurate for the scale-height to be found; but use can be made of the fact that 3·7 km. of the change arises from the equatorial bulge. Of the observed increase in air density by a factor 1·24 ($\pm$ 0·05), a factor 1·08 ($\pm$ 0·02) of this arises from the decrease in perigee distance as calculated from equation 3, so leaving a factor of 1·15 ($\pm$ 0·06) attributable to the 3·7-km. change in height. This leads to a value of 26 ($\pm$ 9) km. for the scale-height.

More accurate orbital data, particularly on the rate of change of period, are needed to take full advantage of the above method. It may then be possible to study the variations of scale-height with latitude.

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[1] Fejer, J. A., *Nature*, **180**, 1413 (1957).
[2] Scott, J. M. C., *Nature*, **180**, 1467 (1957).
[3] Leslie, D. C. M., *Nature*, **181**, 403 (1958).

Application of the Kurie Plot to the Standardization of Pure Beta-Emitters

THE standardization of pure β-emitting nuclides becomes increasingly difficult as the energy of the radiation decreases, because of absorption and scattering effects which occur at the low-energy end of the β-spectrum. It may be possible to circumvent these effects by a technique employed by Waggoner, Moon and Roberts in connexion with a magnetic β-ray spectrometer[1]. This method consists of extrapolating to zero the straight Kurie plot which should be obtained for a single allowed β-ray transition. It is the purpose of this communication to show the relation between the slope of the Kurie plot and the absolute disintegration-rate of a radioactive source,