of the University of California for their assistance and guidance; and the U.S. Weather Bureau for its co-operation.

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The Maximum of a Distribution- or Spectrum-Function

RECENTLY, Bracewell[1] published a paper, in which he explained why the position of the maximum of Planck's energy spectrum depends on whether the frequency or the wave-length is plotted on the horizontal axis. I wish to direct attention to the fact that this is true for all distribution- or spectrum-functions. Though it is indeed obvious that the maximum must move, when the horizontal axis is non-linearly transformed, yet errors have been made in this respect. Zernike[2] pointed out that, for example, an error of this kind was made when Stern and Gerlach[3] described their famous experiments, in which rays of silver atoms were deflected by a magnetic field. They assumed (Fig. 1) that the atoms with the most probable velocity v_p would have the most probable deflexion y_p (thickest part of the precipitation layer). This is only true if the deflexion y and the velocity v of an atom are related by $y = av + b$. However, they are related by $yv^2 =$ constant. The true value of y_p is one-half of the value derived from the last equation with $v = v_p$. This result is due to Semenoff[4], who was apparently the first to recognize the danger, described here. Though geophysicists are particularly vulnerable in this respect, as they have been investigating spectra of all sorts of time-series, yet almost all of them have succeeded in escaping the pitfall.

One exception is Schumann[5], who obtained the spectrum of the daily air pressure on the North Atlantic by a Fourier analysis. He found $T = 5{\cdot}7$ days as the most frequent period in air-pressure. However, when he changed from period to frequency, he found $n = 0{\cdot}033$ (equivalent to $T =$ 30 days) for the maximum of the frequency distribution, a result apparently so strange to him that he looked for an explanation in the wrong direction.

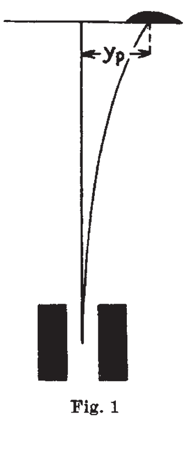

Fig. 1

A group of oceanographers from New York University[6] have studied the energy spectrum of ocean waves. In this case, too, the position of the maximum changes when one considers the distribution of periods instead of the distribution of frequencies.

Spectra of turbulence, especially turbulence of the lower atmosphere, have been analysed by an American group under Panofsky[7] and by an Australian one under Priestley and Swinbank[8]. Both groups have succeeded in avoiding difficulties, though possibly not for reasons inspired by the risk described in Bracewell's paper and here, as is shown by quotations of Panofsky and McCormick (ref. 7, p. 551): "Since meteorological turbulence covers a wide range of frequencies, a logarithmic frequency scale was judged appropriate"; and of Webb (ref. 8, No. 5, p. 7): "It is appropriate to display the results by plotting $n\,F(n)$ which is the fraction of energy per unit fractional increment of frequency. If a logarithmic scale is used for n, the area under a portion of the curve still represents the energy fraction $\int F(n)\mathrm{d}n$."

The use of log n or log λ as abscissa ensures that for most cases the spectrum peak frequency and the spectrum peak wave-length are related by $n_{\text{max.}} \times \lambda_{\text{max.}} = c$. This was proved by Bracewell[1] for the case of $c =$ constant. It is also true when $c = c_0\lambda^k$ (this occurs, for example, with waves in deep water, where $k = \frac{1}{2}$); but it is certainly not valid for cases of, for example, $c = c_0 \exp(-\beta\lambda)$.

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Phase-distribution in and near the Bright Nucleus of an Aberration-free Diffraction Image

IT was shown in the course of a recent investigation[1] that the three-dimensional phase distribution very close to the geometrical focus of an aberration-free diffraction image in light of wave-length λ is substantially the same as that in a parallel beam of wave-length $(1 - \frac{1}{4}\sin^2\alpha)\lambda$, travelling in the direction defined by the principal ray, where $\sin\alpha$ denotes the numerical aperture of the converging pencil.

Let φ and $\tilde{\varphi}$ denote the respective phases, both normalized to $\pi/2$ at the geometrical focus O, of the complex displacements in the converging pencil and in the parallel comparison beam.

When the numerical aperture $\sin\alpha$ is varied, the form of the equiphase surfaces $\varphi =$ const. undergoes a transformation which is more complicated than a simple linear scale-distortion. However, the difference $\varphi - \tilde{\varphi}$ between the phases of the actual and the comparison waves at a point P near focus can be expressed, to a useful approximation, as a function of the two optical parameters u, v alone, where $u/4\pi$ measures the 'number of fringes of defocusing' of P relative to O, and v/π the 'number of fringes of lateral displacement' of P relative to O, as they would be seen

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