

December 29, 1956

Fig. 1. Relationship between operator performance and traffic level

The mathematics involved in solving this problem for automatic exchanges were first developed by Erlang (Brockmeyer, Halstrom and Jensen)<sup>1</sup>. Subsequent extensions<sup>2</sup> provide a basis for calculating subscriber delay, given the traffic-level, the number of operators available to deal with it, and the operator handling time per call. Theoretically, therefore, one can predict how subscriber delay would be affected if the number of operators at an exchange were to be varied, if one can assume that operator time per call is independent of traffic-level. With the cooperation of the General Post Office

this assumption has been tested at a cordless auto-manual exchange by experimentally varying the traffic load per

operator during a four-week period. Operator waiting time was measured by Tippett's<sup>3</sup> snap-Operator reading method, and from it, operator time per call was calculated. The result is shown in Fig. 1.

It will be seen that over the range considered, for all practical purposes operator time per call varies inversely as the logarithm of the traffic load on the operator. An assumption of independence is clearly invalid.

R. CONRAD

 $\{(a -$ 

Medical Research Council Applied Psychology Research Unit, Cambridge. Oct. 24.

<sup>1</sup> Brockmeyer, E., Halstrom, H. L., and Jensen, A., Trans. Danish Acad. Tech. Sci., 2 (1948).

<sup>2</sup> Wilkinson, R. I., Bell System. Tech. J., 32 (1953). <sup>\*</sup> Tippett, L. H. C., J. Textile Inst., 26 (1935).

## **Missing Values in Factorial Experiments**

METHODS of estimating missing values have been discussed by Goulden<sup>1</sup>. Some applications to factorial experiments were presented by Cochran and Cox<sup>2</sup>, but they do not give a general formula. If there is one replication of an  $a \times b \times c \times \ldots f$  factorial experiment, with *n* factors *A*, *B*, *C*,  $\ldots F$  at *a*, *b*, *c*, ... f different levels respectively, then it may be shown by minimizing the highest-order interaction

sum of squares that a suitable estimate, y, of a missing value is given by :

$$\begin{array}{l} (-1)^{n-1} \ (a-1) \ (b-1) \ (c-1) \ \dots \ (f-1)y = \\ (G) - a(A) - b(B) - c(C) \ \dots \ + ab(AB) + ac(AC) + \ \dots \ - abc(ABC) - \ \dots \ \\ \dots \ + (-1)^{n-1} \ bc \ \dots \ f(BC \ \dots \ F) \end{array}$$

where (G) is the sum of the available observations, that is, the incomplete grand total, (A) is the sum of the bc . . . f-1 observations for the level of A which contains the missing value, or the incomplete A total, (AB) is the incomplete  $A \times B$  total, . . . and  $(BC \ldots F)$  is the sum of a-1 observations. There will be  $2^n - 1$  terms on the right-hand side. For two factors this reduces to the well-known randomizedblock formula :

$$(a-1)(b-1)y = a(A) + b(B) - (G)$$

The purpose of this communication is to point out that the above estimate has a property which, except for the  $2 \times 2$  table (see ref. 1, p. 310), has not been recorded. The estimate is identical with that obtained by equating to zero a particular component of the highest-order interaction. This is the component, with one degree of freedom, which measures the interaction of the comparisons of the level of each factor in which there is a missing value, with all other levels of the factor. The relation may be verified in any particular case by expanding the above estimation equation in terms of individual observations. This alternative estimate, which will in general be more laborious to calculate, may be obtained by expanding the function :

1) 
$$A_m - \sum A_i$$
 {  $(b-1)$   $B_n - \sum B_j$  ... { $(f-1)$   $F_t - \sum F_k$  = 0  
 $i \neq m$   $j \neq n$ 

where the product  $A_x B_y C_z \ldots F_w$  represents the observation at levels  $x, y, z, \ldots w$  of the factors; i, j, etc., run from 1 to a, b, etc., respectively; and where observation  $A_m B_n C_o \ldots E_t$  is the one missing.

A more detailed account of these results is being prepared for submission to the New Zealand Journal of Science and Technology.

G. M. WRIGHT

## Crop Research Division,

Department of Scientific and Industrial Research, Christchurch. New Zealand. July 17.

<sup>1</sup> Goulden, C. H., "Methods of Statistical Analysis", 2nd. edit., 318 (Wiley, New York, 1952).
<sup>2</sup> Cochran, W. G., and Cox, G. M., "Experimental Designs", 202 (Wiley, New York, 1950).

## A Mesolithic Burial Tumulus from Ceylon

STONE age burials that have yielded more or less complete skeletons lying in the postures in which they were buried are extremely rare and this is particularly so of Ceylon and India.

Pitted pebble hammers were recorded from Ceylon in 1942<sup>1</sup>, and recently what appears to be a kitchen midden cum burial tumulus yielding more or less complete skeletons of the makers of these artefacts has been discovered at Bellanbandi Palessa at lat. 6° 31' N., long. 80°  $48\frac{1}{2}$ ' E., about two miles west

No. 4548