not very far removed from that of Schläfli, for whom Cayley had a high regard—enough for him to undergo the labour of translating some of Schläfli's papers into English so that they might be published in Britain.

Volume 2 is mainly on algebra and geometry. The particular gem, by which Schläfli's name is perhaps best known, is the theorem (1858) of the 'double six', concerning the intersections of two pairs of six lines in space. This occurs in a memoir on the classification of cubic surfaces, for the double six configuration is closely related to the twenty-seven lines on the cubic surface. But the theorem does not depend on the theory of the cubic surface, and many other proofs have been given ; a melancholy interest attaches to the ingenious proof which relates the result to the celebrated theorem of Miquel on the foci of a set of parabolas, for this was devised by the brilliant young English geometer, Wakeford, shortly before his death on the Western Front in 1916.

Analysis occupies most of the third and final volume. There is, for example, a paper which has the air of being much more modern than its date (1875), in which the asymptotic expansions of the Bessel functions of the third kind are investigated ; Watson remarks that this does not appear to have received the attention it deserves. Another paper of importance deals with spherical harmonics. Heine had unsuccessfully attempted to give appropriate definitions of the general harmonics $P_n^m(\mu)$, $Q_n^m(\mu)$ for complex values of m and n ; Schläfli solved the problem for the zonal harmonics ($m = 0$), by means of single-circuit integrals. The general problem was solved somewhat later, by Barnes and Hobson independently, the former using single-circuit integrals and the latter double-circuit integrals of the Pochhammer type.

In spite of some early struggles, Schläfli seems to have had a quiet and happy life, winning the affections of many generations of pupils, and contributing substantially to the pure mathematics of his time. These three volumes, beautifully produced by Verlag Birkhäuser, and well edited with sound but unobtrusive comments by Kollros, Burckhardt and Hadwiger, are an appropriate memorial to Schläfli's versatile, polished craftsmanship.

T. A. A. Broadbent

PLANETARY NEBULAE

Gaseous Nebulae

By Prof. L. H. Aller. (International Astrophysics Series, Vol. 3.) Pp. xvi+322. (London : Chapman and Hall, Ltd., 1956.) 63*s*. net.

I HAD hoped on opening this book to find that it dealt chiefly with the diffuse gaseous nebulae, which during recent years have become of special interest because of their connexion with interstellar matter and the problems of radio astronomy, with the formation and existence of the hot Population I stars, and through these with important problems of the history and structure of the Milky Way. Although "Gaseous Nebulae" indeed devotes some attention to these objects, its chief concern is with the planetary nebulae, and we are informed that Prof. Lyman Spitzer is at present preparing a book on interstellar matter and the diffuse nebulae. Topical interest in the planetary nebulae (which have nothing to do with planets) is less intense, although for more than half a century they have offered fascinating problems to both theoretical and observational astronomers. They are an order of magnitude smaller than the diffuse nebulae, each is centred on a hot star ; although often complicated, they show regularities of shape approximating to spheroidal shells, or rings, or even helices ; they appear to belong to Population II, and they have no close connexion with interstellar matter.

Nevertheless in some respects, for example, in their spectra, these two distinct classes of nebulae have much in common. Both consist of highly rarefied gas, giving a bright line spectrum, with a considerable number of 'forbidden' lines. The spectrum is produced by a fairly well understood mechanism of ionization, recombination and electron collision, dependent in the first place on dilute ultra-violet radiation from one or more hot stars. But although the general operative principles are known, detailed calculation of the spectrum, or of the physical conditions within the nebula, leads to most formidable complexities. Further, the best observers and the most powerful telescopes are unable with present techniques to give the theorist much of the information which he knows to be of vital importance, and he is too often reduced to forcing a way through his problems by the aid of drastic and not always very convincing simplifications.

Prof. L. H. Aller is a well-known expert in planetary nebulae and here we have a book by an enthusiastic specialist, for specialists. According to the preface, it is based on a much-revised manuscript twelve or more years old. I suspect that the core of the book, Chapter 4 ("Physical Processes in Gaseous Nebulae") and Chapter 5 ("The Forbidden Lines"), which also appears to be the most carefully written part, comes from this manuscript. The other chapters, dealing with types of gaseous nebulae, methods of observation, distances and dimensions, the spectra, the illuminating stars, the structure and internal motions, and the diffuse gaseous nebulae, contain a great amount of very varied and detailed information, too much for clarity, and seem to me to need more thought in arrangement and selection. The specialist, already well acquainted with the subject, may be able to browse with profit, doubtless annotating liberally as he goes. Reading critically and adding much from his own experience, he will realize once more the complexities of the subject, the weaknesses and difficulties of current theories, and the points at which more measurements are urgently needed. He will also find new and hitherto unpublished material, and will be provided with much food for thought.

But it seems to me that the less-advanced student, or the inquirer from another branch of physics, will find complicated matters presented too briefly. He is likely, too, to have difficulty in seeing the wood for the trees and may find himself somewhat bewildered. I cannot recommend the book for such a reader, unless he has a well-informed mentor near at hand to provide a running commentary of elucidation, for there is much which cannot be properly understood without a very wide knowledge of both theoretical and practical astrophysics. Even then, many questions will remain unanswered, some of them pointed out by the author, some not.

There are a number of misprints, including three curious examples on pp. 17, 20 and 229, respectively. Long and useful bibliographies are provided at the end of each chapter.

R. O. Redman