earthquake shocks, or by the roof breaking quite through when very thin, so as to cause two such hollows to unite, or the liquid of any of them to flow out freely over the outer surface of the Earth; or by gradual subsidence of the solid, owing to the thermodynamic melting which portions of it, under intense stress, must experience, according to views recently published by my brother, Professor James Thomson. The results which must follow from this tendency seem sufficiently great and various to account for all that we see at present, and all that we learn from geological investigation, of earthquakes, of upheavals and subsidences of solid, and of eruptions of melted rock."

Lord Kelvin thus seems to have anticipated some of Mr. Gold's ideas, although they, of course, differ as to the origin of the supposed porous structure, and Mr. Gold has given more attention to what such a hypothesis might entail.

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[1] *Nature*, **175**, 575 (1955).
[2] *Trans. Roy. Soc. Edin.*, **23**, 169 (1864); "Mathematical and Physical Papers", **3**, 311.

It is interesting to know that Lord Kelvin held similar views though for different reasons, and especially that he considers also that a porous structure of the Earth would account for a variety of the phenomena that have occurred on the surface. The views of Chamberlain and Moulton, following their planetesimal hypothesis of the formation of the Earth, are perhaps even more similar to those which I have expressed. This set of ideas seems to me to have been much neglected despite the fact that it is in good accord with modern physical and geophysical knowledge.

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Influence of Expanding Space on the Gravitational Fields surrounding Individual Stars

Einstein and Strauss[1] have found solutions of the gravitational field equations of general relativity

$$G_{\mu\nu} = -k\rho v_\mu v_\nu \quad (1)$$

$\mu, \nu = 1, 2, 3, 4$, such that (i) outside a spherical boundary $r = R_1$ (R_1 is a constant), the metric is homogeneous and isotropic, the co-ordinates are co-moving ($v_r = 0$, $r = 1, 2, 3$) and the density ρ does not vanish; (ii) inside $r = R_1$, the metric is spherically symmetric and $\rho = 0$, except for a possible singularity at or near the space-origin.

It is shown that the field inside $r = R_1$ is equivalent to the Schwarzschild field. Einstein and Strauss therefore conclude that "the expansion of space has no influence on the structure of the field surrounding an individual star".

However, Pirani[2] has shown that this negative result is obtained only because of the omission of the cosmological term $\lambda g_{\mu\nu}$ from the left-hand side of equation (1). By introducing this term, he has found that expanding space does influence the gravitational field.

Generally, stars are radiating energy. Hence we considered the Einstein–Strauss problem when the singularity at or near the origin is a source of radiation. In that case the space surrounding this singularity is not empty but is traversed by radially flowing radiation for which

$$G_{\mu\nu} = -k\sigma\omega_\mu\omega_\nu$$
$$\omega_\mu\omega^\mu = 0$$
$$(\omega^\mu)_\nu\omega^\nu = 0$$

σ being the density of flowing radiation[3]. The differential equations for this field can be solved in a manner similar to that employed by Einstein and Strauss in the investigation of the field (ii) mentioned above; but a fundamental difference arises when we try to fit this radiation-field with the pressure-free cosmic field given in (i) above, over a boundary $r = R_1$. The boundary $r = R_1$ now becomes non-static and moves with the velocity of light. The necessary jump conditions to be satisfied at such boundaries are derived by O'Brien and Synge[4]. According to them, if co-ordinates are so chosen that the boundary $r - R_1(t) = 0$ becomes $x^4 = 0$ in the new co-ordinates, then one of the essential conditions is the continuity of T_k^4 at this boundary ($k = 1, 2, 3, 4$). We have found that if the Einstein–Strauss singularity radiates energy, the components T_k^4 (evaluated in the appropriate co-ordinate system) are all zero for $r < R_1(t)$, but two of them are definitely non-zero in the cosmic field $r > R_1(t)$. It is therefore not possible to make T_k^4 continuous across the boundary $r = R_1(t)$. Hence we conclude that if the Einstein–Strauss singularity radiates energy, the field (ii) for $r < R_1(t)$ can never be fitted at the boundary $r = R_1(t)$ to a pressure-free cosmic field (i) with spatially homogeneous non-zero density of matter, irrespective of the vanishing or otherwise of the cosmic 'constant' λ. The jump conditions of O'Brien and Synge allow the fitting of such radiation fields with flat space-time alone, at great distances.

According to Einstein and Strauss, the expanding space of cosmology can be fitted on to a static Schwarzschild field without influencing it. But we have now seen that as soon as the Schwarzschild's singularity begins to radiate energy, the Einstein–Strauss process of fitting the two fields breaks down. We therefore cannot locate a definite (static or non-static) boundary $r = R_1$, such that on one side of it we have expanding space and on the other side a field quite uninfluenced by the surrounding expanding space.

Since all stars are radiating energy, a simple appeal to the jump conditions of O'Brien and Synge leads one to the conclusion that the expansion of space does influence the structure of the fields surrounding individual stars. The exact nature and extent of this influence are now being investigated.

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