Non-Linear Plasma Oscillations and Bursts of Solar Radio Emission

Wild and co-workers[1] have reported observing a fundamental and its second harmonic in the radio spectrum of some solar disturbances. Three characteristic features of the observations are: (i) the frequency bands drift towards the lower frequencies with time; (ii) the received intensities of the fundamental and second harmonic are similar; (iii) the harmonic peak usually occurs at slightly less than twice the frequency of the fundamental peak.

Because of their frequency drift, these bursts of radiation have been associated with the outward travel of streams of particles in the solar atmosphere. Inferred velocities range from several hundred to 10^5 km./sec. Such streams of particles could supply the energy of the most intense radio disturbances.

The observed intensity and frequency ratios have been explained by Wild *et al.* in terms of propagation characteristics of the solar atmosphere. In doing so it was found necessary to assume that the fundamental emission occurs at, or near, the plasma frequency. This points strongly to the origin of the radiation in plasma oscillations. The presence of the second harmonic implies a non-linear oscillator and suggests longitudinal plasma oscillations.

It is the purpose of this communication to show that exact travelling-wave solutions, rich in even harmonics, can be obtained for non-linear plasma oscillations.

The case investigated here is that of longitudinal electron oscillations in a plasma stream. Effects of temperature and magnetic field are neglected.

Let v and n be the electron velocity and density; v_0 and n_0 those of the positive ions in the plasma; E the electric field; ω the angular wave frequency, and ω_0 the angular plasma frequency given by $\omega_0^2 = 4\pi e^2 n_0/m$, $-e$ and m being the electron charge and mass. If the two independent variables, t and x, occur only in the combination $\omega t - kx$, the equations describing the plasma are (in e.s.u.):

$$\text{equation of motion: } v'(\omega - kv) = -\frac{e}{m}E, \quad (1)$$

$$\text{equation of continuity: } n'(\omega - kv) - knv' = 0, \quad (2)$$

$$\text{and Poisson's equation: } n - n_0 = \frac{k}{4\pi e}E'. \quad (3)$$

The prime denotes differentiation with respect to $\omega t - kx$.

Similar equations have been solved by Akhiezer and Lubarskii[2], but their solutions do not seem to satisfy Poisson's equation.

$$\text{Write} \quad u = v - v_0. \quad (4)$$

Assuming that the electron d.c. current density equals that of the positive ions, $n_0 e v_0$, an exact set of solutions of equations (1)–(3) are:

$$\omega = \omega_0 + kv_0, \quad (5)$$

$$\omega t - kx = (l + \tfrac{1}{2})\pi + (-1)^l \frac{u_{\text{max.}}}{\omega_0/k}\sqrt{1 - (u/u_{\text{max.}})^2} + (-1)^l \sin^{-1}(u/u_{\text{max.}}), \quad (6)$$

where $l = 0, \pm 2, \pm 4, \ldots$ applies where u' is positive, and $l = \pm 1, \pm 3, \pm 5, \ldots$ where u' is negative. The origin, $\omega t - kx = 0$, has been placed at $l = 0$, $u/u_{\text{max.}} = -1$.

The electron density and electric field are given in terms of the electron velocity by:

$$\frac{n}{n_0} = \frac{1}{1 - \dfrac{u}{\omega_0/k}} \quad (7)$$

and

$$\frac{E}{E_{\text{max.}}} = (-1)^{(l+1)} \frac{u_{\text{max.}}}{\omega_0/k}\sqrt{1 - (u/u_{\text{max.}})^2} \quad (8)$$

where

$$E_{\text{max.}} = \frac{m\omega_0^2}{ek} \quad (9)$$

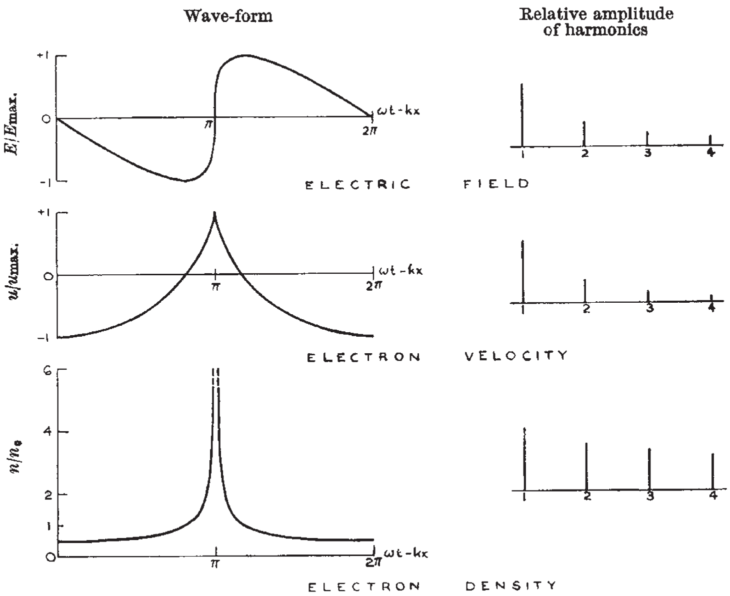

Fig. 1. Longitudinal plasma oscillations. The maximum electron velocity equals the phase velocity

The oscillations reach their greatest amplitude where $u_{\text{max.}} = \omega_0/k$, that is, where the maximum electron velocity, $v_{\text{max.}}$, equals the phase velocity, ω/k. The wave-form and harmonic content of the oscillations for this case are illustrated in Fig. 1. As can be seen, the oscillations contain even and odd harmonics, with about 70 per cent second harmonic in the case of the electron density. Below maximum amplitude the harmonic content is weaker and approaches the linear case (fundamental only) for small amplitudes.

If large oscillations of this kind are responsible for the emission of solar radio bursts, the present results would suggest that harmonics higher than the second may yet be discovered.

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[1] Wild, J. P., Murray, J. D., and Rowe, W. C., *Nature*, **172**, 533 (1953). Wild, J. P., Roberts, J. A., and Murray, J. D., *Nature*, **173**, 532 (1954). Wild, J. P., Murray, J. D., and Rowe, W. C., *Aust. J. Phys.* (in the press).

[2] Akhiezer, A. I., and Lubarskii, G. Y., *C.R. Acad. Sci., U.S.S.R.*, **80**, 193 (1951).