each moving a distance equal to their mean distance apart[1], so that to a first approximation no dislocations cross each other, and relatively few vacancies or interstitial atoms are generated.

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Simplified Particle-size Calculations

PARTICLE-SIZE analysis by microscopical methods provides large samples of measurements which then need numerical analysis to give estimates of average diameters, specific surface, etc. When, as is often the case, particle sizes are log-normally distributed[1-4], tedious summation of powers of the sizes can be avoided if the cumulative distribution is plotted on log-probability paper[1,4]. From this, estimates of the mean, $\bar{y}$, and standard deviation, σ, of the distribution of log sizes can be read off; from these two values can be calculated any of the many kinds of average which are used in this work. If D be such an average, the general equation for this purpose is:

$$\log D = \bar{y} + c\,\sigma^2, \tag{1}$$

where the value of c depends only on the kind of average, D, that is being calculated[1,4].

Even this calculation can be avoided, and a simple completely graphical method substituted with little loss of precision.

If we regard the (linear) abscissa scale (see Fig. 1) as the normal deviate z with origin at the 50 per cent point, the line drawn on probability paper has the equation:

$$Y = \bar{y} + \sigma z. \tag{2}$$

If one unit of y (between particle sizes of 1 and 10, or 10 and 100) occupies a distance A on the paper and one unit of z a distance B (the distance between the 0·1 per cent and 99·9 per cent abscissæ is $6{\cdot}2B$), equation (2) becomes:

$$Y = \bar{Y} + \frac{A}{B}\,\sigma Z, \tag{3}$$

where Y, Z are co-ordinates in units of distance and $\bar{Y}$ is then the ordinate corresponding to $\bar{y}$.

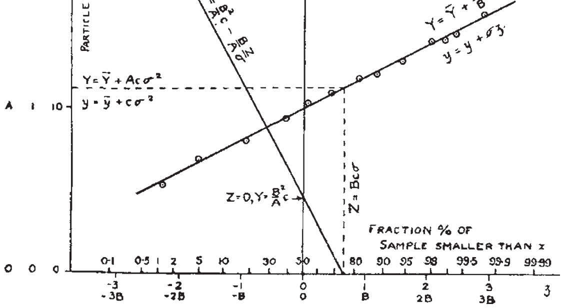

Fig. 1. Graphical method using logarithmic probability paper

The line through the point $Z = 0$, $Y = \frac{B^2}{A}c$ ($z = 0$, $y = \frac{B^2}{A^2}c$) normal to (3) will be:

$$Y = \frac{B^2}{A}c - \frac{B}{A}\frac{Z}{\sigma}, \tag{4}$$

and will meet the Z-axis at $Z = Bc\,\sigma$.

The ordinate of (3) corresponding to this value of Z, namely,

$$Y = \bar{Y} + Ac\,\sigma^2,$$

is equivalent to: $y = \bar{y} + c\,\sigma^2 = \log D$, and gives the average that is sought.

The scale-factors A, B are constant for any one kind of probability paper, and c depends only on the average diameter being estimated. Hence the point $Z = 0$, $Y = \frac{B^2}{A}c$ is independent of the sample of sizes and can be determined beforehand for each average diameter.

There are occasions also when very approximate results will suffice and are needed rapidly; the number of particles measured and the calculation can then both be greatly reduced. It is necessary only to pick out and measure the largest particle and the smallest particle of the sample and to make a rough estimate of the total number, n, of particles that could have been measured.

If y_1, y_n are the logarithms of the largest and smallest particle sizes, their mid-range $\frac{1}{2}(y_1 + y_n)$ provides an estimate of the mean $\bar{y}$, and from the range the standard deviation σ can be estimated as $\frac{1}{w}(y_1 - y_n)$, where w is roughly the expected range in samples of n. The value of w depends on n and can be chosen from the following table, which is based on Tippett's table[5] and is sufficient for this approximate method.

n	w
20–35	4
35–70	$4\frac{1}{2}$
70–140	5
140–300	$5\frac{1}{2}$
300–700	6
700–1,700	$6\frac{1}{2}$
1,700–3,000	7

It must be borne in mind not only that this method is approximate and not efficient but that, as in all cases where range and mid-range are used as estimators, the results can be affected, without it being known, either by truncation of the sample or by the presence in the sample of exceptionally large or small particles which do not belong to the population of particles under consideration. Thus it is recommended that this method is best used as a rough check on calculations by some longer method, or for obtaining quickly approximate information from a sample of a material about which something is already known, so that an unexpectedly large or small estimate of σ can be recognized and regarded with suspicion.

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