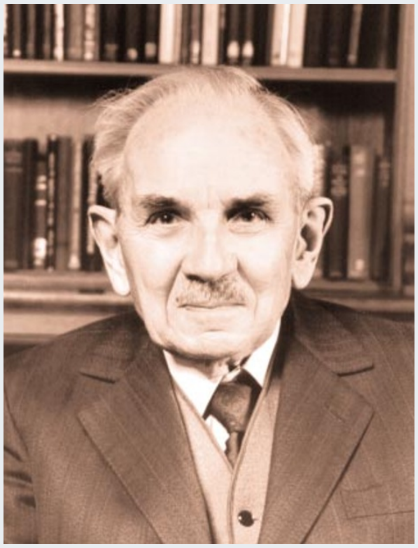
Obituary

Jean Leray (1906–98)

Master of applied mathematics



Only a few months after the death of André Weil, whose work and influence was described here on 29 October last year (*Nature* 395, 848; 1998), another major figure has left the mathematical scene. Jean Leray died on 10 November, at La Baule in France, after a long career spent mostly as a professor at the Collège de France.

Weil and Leray had curiously parallel lives: both were born in 1906, both were educated at the Ecole Normale Supérieure in Paris, and both died in 1998. Weil left France during the Second World War and spent the remainder of his career in Princeton, while Leray was captured with his combat unit in 1940 and was detained in Austria until the end of the war, when he returned to France. They symbolize the two opposing trends that have characterized mathematics over the past 50 years.

Leray viewed mathematics as a tool for modelling, and drew his inspiration from problems in mechanics and physics, such as fluid dynamics and wave propagation. He was fond of explaining how the road from mathematics to applications is two-way, and how a purely mathematical theorem (concerning, for instance, the existence and uniqueness of solutions of systems of partial differential equations) might have profound physical implications.

Leray's studies of the Navier–Stokes equations, which are fundamental to fluid dynamics, were of this kind. In an epoch-making paper of 1934, published in *Acta Mathematica*, he showed that smooth initial data may give rise to non-stationary solutions which remain smooth for a finite time only, after which they become irregular (the maximum velocity 'blows up'), and satisfy the Navier–Stokes equations only in a generalized sense; that is, such solutions do not satisfy the equations at every point, but must be integrated against smooth test functions. Leray termed such solutions 'turbulent'. In the same paper, he proved that regular solutions are uniquely determined by their initial data, and suggested that, on the other hand, there might be many 'turbulent' solutions corresponding to the same initial data. This conjecture remains unproved, and is one of the competing arguments for describing the onset of turbulence.

Weil, on the other hand, viewed mathematics as a structure developing according to its own needs, and prided himself on never having done anything applied. He was one of the founders of the Bourbaki group, which set itself the task of rewriting mathematics on firm premises, free from all compromises with common sense and so-called geometrical or physical intuition. This line of thinking was to be enormously influential in France, where it came to dominate teaching and research until the mid-1970s. Leray kept away from the crowd and pursued his own brand of mathematics, firm in his conviction that it was the right one, and that it would prevail. I still remember his lectures. He was a mild-mannered, dapper man with a grey moustache, who squinted at his audience and lost it rather quickly; but he continued to write on the blackboard amidst a respectful silence, confident that the mathematics were there for all to see and needed no further explanation.

By great irony, Leray himself turned out to be one of the main sources for the Bourbaki brand of mathematics. During the five years he spent as a prisoner of war in OFLAG XVIIA, he not only helped found a 'university' but also pursued his own research. He was however an ardent patriot, and he did not want the Germans to discover his competence in fluid mechanics, lest he be compelled to contribute to their war effort. So he turned to 'useless' mathematics, namely algebraic topology, the study of algebraic structures associated with topological spaces.

The topic did not catch the interest of German intelligence. But it has become central to modern geometry and physics, because in both cases one is concerned with symmetries, that is, with the way groups can act on an underlying space: the description of these actions, and the way the orbits fit in to reconstruct the whole space, are problems that cannot be tackled without the tools of algebraic topology. Two of these tools are products of Leray's war years: the 'spectral sequence', which has been central in the investigation of homotopy groups of spheres (the different ways to map a sphere into a sphere of lower dimension); and the 'theory of sheaves', which is fundamental to many areas of pure mathematics, such as the study of functions of several complex variables.

After the war, Leray returned to the study of partial differential equations, and remained highly active until his death. Three volumes of his selected papers, edited by Paul Malliavin, were published in 1997 by Springer-Verlag and the Société Mathématique de France, and the introductions to each volume give more detail about his work. Here, I would like to stress Jean Leray's modernity. His 1934 paper on the Navier–Stokes equations was truly 40 years ahead of its time, and is unsurpassed even today. He solved nonlinear systems of partial differential equations at a time when the tools for solving linear ones had yet to be invented. Not until the 1960s did the mathematical world master the notion of weak solutions, Leray's 'turbulent' solutions, and not until the 1970s were his methods for solving nonlinear systems fully understood.

Leray was so far ahead of his time because of his tremendous technical capability and geometrical insight. In his hands, energy estimates for partial differential equations became combined with ideas from algebraic topology (such as fixed-points theorems) in a highly original combination which cracked open the toughest of problems. He was the first to adopt the modern point of view, whereby a function is not a complicated relation between two sets of variables, but a point in some infinite-dimensional space; and he was the first to solve problems in functional calculus (and hence, partial differential equations) by using the geometry of such spaces. All of these ideas are classics, and little has since been added to them. If we define algebra as the study of abstract mathematical structures, and analysis as solving equations arising from physics or other sciences, then Weil can be said to have been the first modern algebraist and Leray the first modern analyst.

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