

for various time-intervals. The results are shown in Fig. 2, where time-intervals are recorded on a logarithmic scale. The graphs show that the total deposit increases most rapidly with time during inversions.

Comparison of results from the two methods of assessment showed that deposits collected beneath the aircraft track had solution strengths which were approximately double that of the original solution, that is, 50 per cent of the solvent had been lost during the very few seconds required for these deposits to form. A correlation test showed that the loss of solvent increased significantly with time-interval, but so slowly that after 3-4 min. the losses had only increased to 60 per cent. It seems probable that the kerosene evaporates very rapidly during the first few seconds after the droplets have been formed, leaving droplets composed of relatively involatile substances.

The results suggest that a simple, practicable relationship can be found between aerosol behaviour and simple meteorological measurements. The exact form of the behaviour of an aerosol will depend greatly upon the sizes of its constituent droplets, and our results are being analysed to discover how droplets of various sizes are deposited. A preliminary study suggests that the smaller the droplet the greater is the effect of a change in the degree of atmospheric turbulence, as would be expected. Experiments are also being carried out with different heights of the aircraft, and over areas of woodland.

A fuller account of this work is to be published elsewhere.

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Longitudinal Elastic Waves in Cylindrical Rods

ALTHOUGH Pochhammer's¹ well-known solution for elastic waves in cylindrical rods has been in existence for many years, two problems arising out of it in connexion with longitudinal motion have yet to be solved.

The first difficulty relates to the physical significance of the higher modes (as opposed to their theoretical importance, which has been explained by Cooper²). The transmission of a pulse in one of the higher modes would be evidenced by very high phase velocities for long waves and, in the course of carefully planned experiments with a pressure bar, Davies³ did not detect such an effect (see, however, Morse⁴, who describes experiments with rectangular rods). An explanation of this fact would be reassuring, since engineering theories of longitudinal waves do not give rise to higher modes⁵.

The second problem is that pointed out by Love⁶, and concerns a free end of the cylinder where reflexion occurs. The stresses of Pochhammer's solution may be written in the form:

$$\begin{aligned} \mathbf{xx} &= (\text{real function of } r) \exp\{i \xi (x - ct)\} \\ \mathbf{xr} &= (\text{imaginary function of } r) \exp\{i \xi (x - ct)\} \end{aligned} \quad (1)$$

for any particular wave number ξ and phase propagation speed c . Thus, if the normal stress \mathbf{xx} is always to vanish at a free end, \mathbf{xr} will not do so.

This problem may be regarded as if perfect reflexion occurs with the superimposed effects of an applied stress system:

$$\mathbf{xx} = 0; \quad \mathbf{xr} = (\text{function of } r) \exp(i \xi ct) \quad (2)$$

at the hitherto free end. Now the question arises: Is the effect of the auxiliary system (2) local, or does it, in some way, distort the retreating reflected wave? It is, perhaps, to be expected that the effect is local since steady longitudinal vibrations of a rod might not otherwise be possible; but it would be virtually impossible to find the answer by direct experiment.

Pochhammer's results are reached if the trial solution:

$$\begin{aligned} u_x &= f_1(r) \exp\{i \xi (x - ct)\}; \quad u_r = f_2(r) \exp\{i \xi (x - ct)\}; \\ u_\theta &= 0 \end{aligned} \quad (3)$$

is used in the general equations of motion. If equations (3) embrace the most general longitudinal wave and they inevitably lead to equations (1), it is apparent that a motion giving (2) is not possible. This is obviously so if, in fact, the only physically realized mode of Pochhammer's solution is the first.

What is the alternative if displacements due to system (2) are not propagated? Since the applied force always has zero resultant, the problem is analogous to those involving an appeal to St. Venant's principle, the disturbance now being time-dependent. It has been shown by Goodier⁷ that the effects would be local in the static case as a simple consequence of the conservation of energy. In the absence of wave transmission, this would still be true for dynamical cases. Thus a harmonic wave approaching a free end would suffer complete reflexion. But its form would become temporarily blurred near the stress-free section.

These views are advanced as a suggestion rather than as a proof. They are offered because it appears to me that this is not the only vibration problem the exact solution of which could be completed by the establishment of some form of dynamical St. Venant principle; for example, certain plate problems seem to fall into this category.

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¹ Pochhammer, L., *J. reine u. angew. Math.*, **81**, 324 (1887). See also ref. 4.

² Cooper, J. L. B., *Phil. Mag.*, (7), **38**, 1 (1947).

³ Davies, R. M., *Phil. Trans.*, A, **240**, 375 (1946).

⁴ Morse, R. W., *J. Acoust. Soc. Amer.*, **20**, 833 (1948).

⁵ Bishop, R. E. D., *Aero. Quart.*, **3**, 280 (1952).

⁶ Love, A. E. H., "The Mathematical Theory of Elasticity", 201 (1927).

⁷ Goodier, J. N., *Phil. Mag.*, (7), **8**, 607 (1937).

Elastic Deformation and the Laws of Friction

It has been shown by Bowden and his co-workers¹ that the classical law of friction $F = \mu W$ is dependent on plastic deformation of the asperities in contact on the rubbing surfaces. In the case of high polymers, however, Lincoln² has shown that, ideally, relationships of the form $F = kW^{2/3}$ are to be expected at small loads, as a result of elastic deformation of the asperities. Other examples of high polymer friction³, and particularly the friction of textile fibres^{4,5}, indicate similar relationships although, in the case