

to criticize the allocations which the authors have made.

More serious is the criticism which must be made of inaccuracies and misleading or vague statements. A student might well have misgivings when he discovered that Fehling's test on a non-reducing sugar failed, although he had followed the instructions to the letter, for there are no directions that the solution should be neutralized after hydrolysis. The descriptions of *Pandorina*, *Volvox* and *Haematococcus* are not entirely accurate, while that of the dehiscence of a fern sporangium scarcely gives a good picture of the course of events. Transfusion tracheids are referred to as parenchyma; laburnum wood is not the happiest choice for tyloses; nor will the student find many chloroplasts in *Nostoc*; and it is surely time that meiosis lost its heterotypic and homotypic divisions. To expect a book of this type to be free from errors or to attain perfect lucidity would be unreasonable; but it does seem reasonable to ask if what might be regarded as a permissible margin—and the examples just noted are only a sample—has not been exceeded.

Nearly all the illustrations are from photographs, and they have been very sparingly used to illustrate material which is difficult to obtain or to prepare. As a whole, they are excellent and form an attractive feature of the book.

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MODERN THEORIES OF INTEGRATION

The Lebesgue Integral

By J. C. Burkill. (Cambridge Tracts in Mathematics and Mathematical Physics, No. 40.) Pp. viii+87. (Cambridge: At the University Press, 1951.) 12s. 6d. net.

The Theory of Functions of a Real Variable

By Prof. R. L. Jeffery. (Mathematical Expositions, No. 6.) Pp. xiii+232. (Toronto: University of Toronto Press; London: Oxford University Press, 1951.) 45s. net.

Lezioni sulla teoria moderna dell'integrazione

Per Prof. Mauro Picone e Tullio Viola. (Manuali Einaudi, Serie di matematica.) Pp. 404. (Torino: Edizioni Scientifiche Einaudi, 1952.) 5,000 lire.

EVERY honours student of mathematics should know why Lebesgue's definition of an integral is preferable to the older definition due to Riemann. One of the easiest ways to acquire this knowledge is to study the clear and concise exposition of the subject by Dr. J. C. Burkill. The preface points out that by adopting Lebesgue's definition we can arrive quickly at results which otherwise require tiresome discussions. This general remark is illustrated by reference to a particular example. There are six chapters, entitled respectively sets of points; measure; the Lebesgue integral; differentiation and integration; further properties of the integral; and the Lebesgue-Stieltjes integral. The tract concludes with hints for the solution of all but the easiest of the fifty-five examples contained in various places, with more detailed solutions of the most important. Perhaps on some pages the virtue of conciseness has been carried to excess. For example, on pp. 7 and 11 there are results without proof, preceded by the statement "the reader will satisfy himself that . . ."; and on p. 10 the proof of a theorem concerning

measure ends curtly with the words, "taking the limit, we have the general case". The table of contents is also too concise. For example, it does not reveal that the book contains the inequalities of Hölder and Minkowski (pp. 65-66), and there is no index. But these are small defects in an excellent book.

A fuller treatment may be found in the book by Prof. R. L. Jeffery, which seems to consist of two distinct parts. The first part is for beginners in the subject, and starts with an introduction giving a brief account of the real number system. Then follow five chapters which may be described roughly as an amplified treatment of the corresponding chapters in Dr. Burkill's book. One novelty is that the treatment of measure is based upon Lebesgue's outer measure, which Prof. Jeffery prefers to call "metric of a set", and denotes by an unusual symbol. No use is made of inner measure, which he considers to be of purely historical interest. Another novelty is the inclusion of G. D. Birkhoff's ergodic theorem, which arose from problems in dynamics and statistical mechanics but is really a theorem in Lebesgue integration.

The second part of the book, which Prof. Jeffery himself considers of greater importance, consists of Chapters 6 and 7, entitled respectively the inversion of derivatives, and derived numbers and derivatives. These are written for more advanced students and aim at giving an exhaustive account of a subject to which the author has contributed by his own researches.

The last chapter, dealing with the Stieltjes integral, forms a sort of appendix to both parts. It starts with a simple account of the Riemann-Stieltjes integral and then proceeds to the more difficult topic of linear functionals and the recent ideas on measure and distribution due to L. Schwartz.

The book contains more than a hundred problems, a bibliography of seventy items, an index of subjects and an index of authors. It is carefully written in an attractive style.

The characteristic feature of the Italian book by Profs. M. Picone and T. Viola is the great emphasis on the Stieltjes integral. The introduction deals with two physical problems, the attraction of a body at an external point and the motion produced by an impulse. The first of these problems leads to an integral which involves elements of mass. Such integrals, in the light of modern physical views on the structure of matter, can be understood only as Stieltjes integrals. The authors then develop the whole theory of integration from this point of view, and introduce the term *massa elementare* for an additive function of finite variation. The word *massa* is used continually, roughly as often as the word *measure* in other treatises, but the word *misura* (measure) seems to occur very seldom. The early parts of the book make use of the original work on the theory of limits due to Prof. Picone. These opening chapters are rather different from those of other books, but later on we find the usual theorems, sometimes approached from an unusual angle. The advanced student will find this book interesting and valuable, containing many novelties which would require too much space to specify in detail. It concludes with four indexes, those of notations, of 'examples' (really applications of the theory), of authors, and of subjects. The style is readable, and the printing and paper are very good indeed.

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