becomes less stringent. A better measure of the characteristic is the slope of the plateau, and this is plotted against pressure of contaminant, for air, in the accompanying graph. The effect is approximately linear for small pressures of contaminant, but appears to reach saturation at about 1 cm. pressure of air. This presumably occurs when all the argon ions moving away from the anode undergo exchange of charge with oxygen or nitrogen atoms. The plateaux were originally about 120 volts long, disappearing completely when the partial pressure of air was about 1 cm. No permanent contamination effects were observed, a plateau comparable, though not identical, with the original, being observed on re-filling.

The effect of carbon monoxide was also observed, since this is a likely contaminant in counters the cases of which are made of cast metal. The effects on the length and slope of the plateau were remarkable, a pressure of 0·5 mm. of carbon monoxide being approximately equivalent to 6·0 mm. of air. The plateau disappeared completely at a pressure of 1·0 mm. of carbon monoxide.

The deterioration of methane-filled counters has been attributed by Farmer and Brown[4] to the deposition of heavy hydrocarbons on the electrodes. It is possible that this effect is enhanced in argon-alcohol counters by the formation of carbon monoxide in the gas.

D. KEEFE
T. E. NEVIN
University College,
Dublin.
C. B. A. McCUSKER
N. A. PORTER
Dublin Institute for Advanced Studies.
June 18.

[1] Spatz, W. D. B., *Phys. Rev.*, **64**, 236 (1943).
[2] Korff, S. A., and Present, R. D., *Phys. Rev.*, **65**, 274 (1944).
[3] Fenton, A. G., and Fuller, E. W., *Proc. Phys. Soc.*, A, **62**, 32 (1949).
[4] Farmer, E. C., and Brown, S. C., *Phys. Rev.*, **74**, 902 (1948).

Effect of Variation in Diameter on the Torsional Vibration of Bars

DURING measurements of the resonant frequencies of torsional vibrations in cylindrical bars, it was observed that variation in the diameter of the bar along its axis caused the resonant frequencies to depart from a harmonic series. In order to relate the resonant frequencies to the shape of the bar, we have adapted a method described by P. M. Morse[1] for calculating the modes of vibrating strings in which the tension varies from point to point along the length.

The method consists in expressing the displacement of the bar at any instant as the sum of a series of normal modes for a uniform bar, and regarding the departure from uniformity as a small perturbation function imposed on the mean bar diameter. Substituting in the wave equation for a non-uniform bar:

$$\frac{\partial}{\partial x}\left(a^4\frac{\partial\theta}{\partial x}\right) = \frac{a^4}{v_0^2}\frac{\partial^2\theta}{\partial t^2},$$

where a is the bar diameter, x the distance along the axis, θ the angular displacement and v_0 the velocity of shear-waves in the material, and dropping out second and higher order terms, we obtain an approximate equation from which the coefficients of the series representing the bar displacement and also the modification to the resonant frequencies can be determined by a process analogous to Fourier analysis.

The result obtained in this way shows that, if the departure of the bar from uniformity is expressed as

$$a = a_0\left[1 + \sum_{p=1}^{\infty} K_p \cos 2\pi px/l + \sum_{p=1}^{\infty} C_p \sin 2\pi px/l\right],$$

then the fractional change in the frequency of the mth harmonic is $-2K_m$. It will be observed that to the order of accuracy of this analysis the frequencies are unaffected by the sine components of the series. Thus, for example, a bar having a uniform taper along its length will have the same torsional resonant frequencies as a uniform cylindrical bar.

In order to verify this result, we measured the resonant frequencies on a bar of aluminium approximately 4 in. long by ½ in. diameter, the diameter being reduced abruptly between points $l/4$ and $3l/4$ distant from one end, where l is the length of the bar, to a value 4 per cent less than that at the ends of the bar. The coefficients of the series representing the radius of the bar are then given by:

$$\begin{aligned} K_p &= (-1)^{(p-1)/2}\ 0{\cdot}16/\pi p \text{ for } p \text{ odd} \\ &= 0 \text{ for } p \text{ even.} \end{aligned}$$

Table 1 shows the observed and calculated values of percentage frequency deviation from the mean value.

Table 1

Harmonic No.	Observed frequency deviation (per cent)	Calculated frequency deviation (per cent)
1	−5·1	−5·1
2	0·0	0·0
3	+1·8	+1·7
4	0·0	0·0
5	−1·0	−1·0
6	0·0	0·0
7	+0·80	+0·73
8	0·0	0·0

A similar analysis of the effect on longitudinal waves predicts half the above deviations, since a enters the wave equation to the second power instead of to the fourth as in torsional vibration; but such an analysis ignores the effect of lateral inertia, which modifies the vibrations by an increasing amount as the ratio of bar diameter to wave-length (λ) increases[2-4]. Measurements confirm this divergence between simple theory and practice, as Table 2 shows.

Table 2

Harmonic No.	$\frac{2a}{\lambda}$	Observed frequency deviation (per cent)	Calculated frequency deviation (per cent)
1	0·061	−2·6	−2·6
2	0·123	0·0	0·0
3	0·184	+0·9	+0·9
4	0·245	+0·1	0·0
5	0·307	−0·4	−0·5
6	0·368	0·0	0·0
7	0·429	+0·2	+0·4
8	0·491	0·0	0·0

H. PURSEY
National Physical Laboratory,
Teddington, Middlesex.
Feb. 20.

[1] Morse, P. M., "Vibration and Sound", 2nd edit., 122 (McGraw-Hill, 1948).
[2] Love, A. E. H., "The Mathematical Theory of Elasticity," 4th edit., 289 (Dover Publications, 1944).
[3] Pochhammer, L., *J.C. Math.* (*Crelle*), **81**, 324 (1876).
[4] Bancroft, D., *Phys. Rev.*, **59**, 588 (1941).