potent an anæsthetic than, amylobarbitone, shows an activity of only 42·5 on the ganglion. The relationship between ganglionic and central nervous depressant activities is expressed in the third column as the ratio A/B. The highest ratio (1·40) is given by butobarbitone.

Other barbiturates, which are not in current clinical use, are now being investigated with the view of determining the structure – action relationships responsible for conferring increased ganglion depressant effects.

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Rigid-Body Motions in Special Relativity

The rigid-body motions proposed by Born[1] and Herglotz[2] have, in general, only three degrees of freedom, so that the world line of one point determines the complete motion. My purpose here is to define a group of rigid-body motions with six degrees of freedom as in Newtonian mechanics. Any such definition must be regarded as tentative, and its physical significance tested by experiment (see following communication from Prof. J. L. Synge).

In the space-time of Minkowski, where a point has the four co-ordinates x^1, x^2, x^3, x^4 ($= ict$), let a world line be determined by the equations:

$$x^r = a^r + b_1\mu_1{}^r + b_2\mu_2{}^r + b_3\mu_3{}^r + b\lambda^r, \quad (r = 1, 2, 3, 4). \qquad (1)$$

Here a^1, a^2, a^3 are three arbitrary functions of a parameter u, and $a^4 = icu$; λ^r is the unit 4-vector in the direction of the da^r/du; $\mu_1{}^r$, $\mu_2{}^r$, $\mu_3{}^r$ form an orthonormal triad of 4-vectors orthogonal to λ^r, which triad may be specified by three arbitrary functions of u giving, say, its Eulerian angles; b_1, b_2, b_3 are constants and $b = (b_1{}^2 + b_2{}^2 + b_3{}^2)^{1/2}$. By letting b_1, b_2, b_3 have a range of values, we get a congruence of world lines depending on six arbitrary functions of u. If a body moves so that the world lines of all the particles form such a congruence, then we shall say that the body has a rigid motion.

The curve $x^r = a^r$ is the history of the particle for which the b's vanish; this particle, which takes a special role, may be called the 'drag-point'. Since $a^4 = icu$, the parameter u may be interpreted as the time along the world line of the drag-point.

The connexion with Newtonian rigidity and the Lorentz contraction may be seen as follows. If x^r and y^r are two events of the congruence with the same value of u, then the invariant $(x^r - y^r)$ $(x^r - y^r)$ with r summed, which is the square of the Minkowski distance, is independent of u and therefore remains constant throughout every rigid motion. The value of $\lambda^r(x^r - y^r)$ also remains constant. This latter condition implies that whenever the body, initially at rest, is accelerated and finally moves in uniform translation, it is shortened in the direction of motion by the Fitzgerald–Lorentz contraction factor.

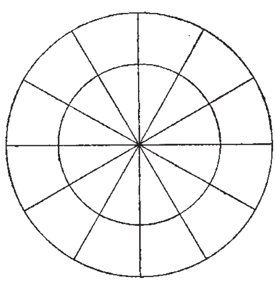

Fig. 1

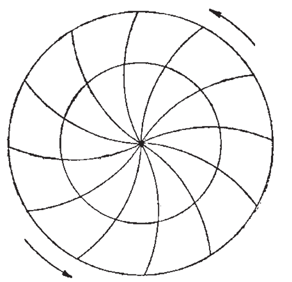

Fig. 2

The following example of rigid motion is a relativistic analogue of the Newtonian rotating disk. Choose for the drag-point the origin of a Galileian observer S so that a^1, a^2, a^3 are all zero. For the rotating triad take $\mu_1{}^r = (\cos\theta, \sin\theta, 0, 0)$, $\mu_2{}^r = (-\sin\theta, \cos\theta, 0, 0)$ and $\mu_3{}^r = (0, 0, 1, 0)$, where θ is some function of u. With b_3 equal to zero, equations (1) give the motion of all points in the x_1, x_2 plane, the first and second derivatives of $\theta(u)$ representing the angular velocity and angular acceleration of the body respectively. Fig. 1 shows the disk at rest in S with radial lines and circles about the origin etched on its surface, and Fig. 2 shows the same etchings as they appear to S when the disk has acquired a constant angular velocity ω, drawn for the case when the peripheral velocity is $\frac{1}{2}c$. The straight radial lines become curved with polar equation $r\omega = c(\theta + \text{constant})$. The radii of the circles remain the same, as indeed is true even in non-uniform angular motion.

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Gardner's Hypothesis and the Michelson–Morley Experiment

When Gardner's hypothesis (see preceding communication) is applied to the Michelson–Morley experiment, performed with an interferometer with horizontal rigid arms, carried on the rotating earth, it yields the following formula for fringe-shifts if we take the drag-point to be the centre of the earth:

$$\delta/w = (D/\lambda)\,(E/c)\,\varphi \sin 2\theta.$$

Here δ is the fringe-shift, w the fringe-width, D the arm-length of the interferometer, λ the wave-length used, E the velocity of the earth's surface due to the earth's rotation alone, c the velocity of light, φ the deviation of the plumb line, and θ the azimuth of the telescope arm, from the north eastward. If, on the other hand, the drag-point is near the interferometer, the predicted fringe-shift is below the limits of observation.

In a set of experiments involving a very great number of observations, D. C. Miller[1] observed fringe-shifts resembling those predicted by the above formula, which reads, when we insert the numerical values appropriate to Miller's experiments:

$$\delta/w = 0{\cdot}23 \sin 2\theta.$$