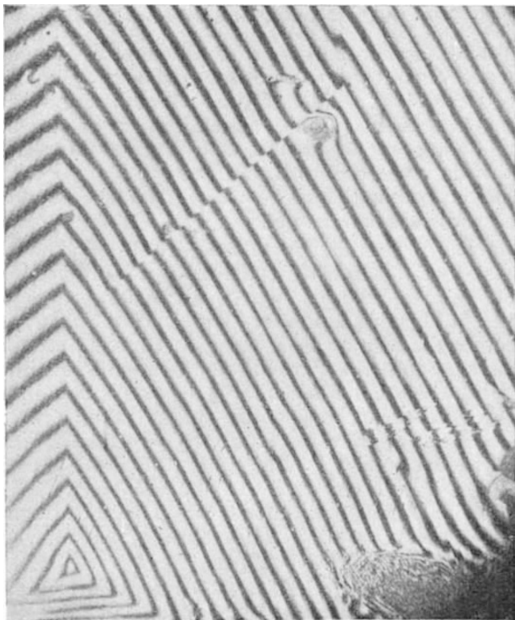

Fig. 2. (× 7)

I wish to thank Prof. S. Tolansky for his continual interest in this work; the Director, Signals Research and Development Establishment, Christchurch, for the loan of the crystal; and the Chief Scientist, Ministry of Supply, for permission to publish this account.

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[1] Amelinckx, S., *Nature*, **169**, 580 (1952).
[2] Nomoto, O., *Nature*, **164**, 359 (1949).

The Energy Momentum Tensor in Dirac's New Electromagnetic Theory

DIRAC has recently[1] proposed a generalization of his new theory of electromagnetism so as to permit vortical streams of the electrical charge. The vortical motion has been introduced by redefining the potentials in a more general manner. Dirac has then framed two stationary principles, both of which lead back to the Lorentz equation of motion for an electric charge, with, however, the ratio, $m/e = k$, only occurring in it. The first of these stationary principles has for its action density

$$L = -\tfrac{1}{4}F_{\mu\nu}F_{\mu\nu} + \tfrac{1}{2}\lambda\,(v_\nu v_\nu - 1), \tag{1}$$

where the potentials are assumed to be of the form

$$A_\mu = kv_\mu + \xi\frac{\partial\eta}{\partial x_\mu}. \tag{2}$$

In these equations v_μ is the velocity vector, $v_\mu v_\mu = 1$, and ξ and η are two independent functions of the x_ν. λ is an arbitrary function of the x_ν. Varying the integral Lagrangian (1), one gets the equations,

$$v_\nu v_\nu = 1, \tag{3}$$

$$k\frac{\partial F_{\mu\nu}}{\partial x_\nu} = \lambda v_\mu \tag{4}$$

and

$$\frac{\partial F_{\mu\nu}}{\partial x_\nu}\frac{\partial\xi}{\partial x_\mu} = \frac{\partial F_{\mu\nu}}{\partial x_\nu}\frac{\partial\eta}{\partial x_\mu} = 0, \tag{5}$$

so that

$$v_\mu\frac{\partial\xi}{\partial x_\mu} = v_\mu\frac{\partial\eta}{\partial x_\mu} = 0. \tag{6}$$

(4) is the Maxwell equation, and (5) after reduction becomes the Lorentz equation

$$k\frac{\mathrm{d}v_\mu}{\mathrm{d}s} = v_\nu F_{\nu\mu}. \tag{7}$$

In making the variation, the A_μ, ξ, η and λ were treated as the independent field functions.

The canonical energy momentum tensor $T_{\rho\sigma}$ derived in the standard way from the Lagrangian (1) is

$$T_{\rho\sigma} = \frac{\partial kv_\lambda}{\partial x_\sigma}F_{\rho\lambda} + \frac{\partial\xi}{\partial x_\sigma}F_{\rho\nu}\frac{\partial\eta}{\partial x_\nu} + \frac{\partial\eta}{\partial x_\sigma}F_{\mu\rho}\frac{\partial\xi}{\partial x_\mu} + \delta_{\rho\sigma}L. \tag{8}$$

This tensor is not symmetrical and so with it angular momentum would not be conserved. To make it symmetrical we may proceed in the usual way (see, for example, Wentzel, "Quantum Theory of Fields", Appendix by Jauch) and add,

$$T'_{\rho\sigma} = -\frac{\partial}{\partial x_\lambda}(kv_\sigma F_{\rho\lambda}) = -\frac{\partial}{\partial x_\lambda}G_{\rho\lambda\sigma} = \frac{\partial}{\partial x_\lambda}G_{\lambda\rho\sigma}. \tag{9}$$

$T'_{\rho\sigma}$ has the necessary properties of vanishing divergence and vanishing integrals of the energy and momentum components, for

$$\frac{\partial T'_{\rho\sigma}}{\partial x_\rho} = -\frac{\partial}{\partial x_\rho}\frac{\partial}{x_\lambda}G_{\rho\lambda\sigma} = 0,$$

$$\int T'_{4\sigma}\,\mathrm{d}x_1\mathrm{d}x_2\mathrm{d}x_3 = -\int\frac{\partial}{\partial x_\lambda}G_{4\lambda\sigma}\,\mathrm{d}x_1\mathrm{d}x_2\mathrm{d}x_3 = -\int\frac{\partial}{\partial x_k}G_{4k\sigma}\,\mathrm{d}x_1\mathrm{d}x_2\mathrm{d}x_3 = 0.$$

Now by (4),

$$T'_{\rho\sigma} = -\frac{\partial kv_\sigma}{\partial x_\lambda}F_{\rho\lambda} - kv_\sigma\frac{\partial F_{\rho\lambda}}{\partial x_\lambda} = \frac{\partial kv_\sigma}{\partial x_\lambda}F_{\rho\lambda} - v_\sigma\lambda v_\rho. \tag{10}$$

So the symmetrical tensor, $\theta_{\rho\sigma} = \theta_{\sigma\rho}$, is

$$\theta_{\rho\sigma} = T_{\rho\sigma} + T'_{\rho\sigma} = F_{\sigma\lambda}F_{\rho\lambda} - v_\sigma\lambda v_\rho + \delta_{\rho\sigma}L. \tag{11}$$

The ξ and η variables do not appear explicitly in this formula, and the tensor is, apart from notation, of the same form as the tensor of the irrotational theory, the θ_{00} and θ_{0k} components of which have been given by Le Couteur[2].

I thank Prof. P. A. M. Dirac for suggesting this problem to me.

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[1] Dirac, P. A. M., *Proc. Roy. Soc.*, A, **209**, 291 (1951); A, **212**, 303 (1952).
[2] Le Couteur, K. J., *Nature*, **169**, 146 (1952).