LETTERS TO THE EDITORS

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Is there an Æther ?

IN his recent communication[1] Prof. Dirac has said that in his new formulation of electrodynamics[2] a preferred motion exists at each point of space.

A preferred motion is also given at each point of space by cosmological observations (apparent isotropy of distant red-shift effects).

It is of interest whether any local physical effects are associated with this cosmologically preferred state of motion, and the analogous question can be asked with respect to Dirac's formulation. We have argued[3] that the first question may be answered in terms of the theory of continual creation (the steady-state theory of the universe), where the cosmologically preferred motion is identified with the velocity of newly created particles. Dirac answers the second question by saying that a small charge placed into a vacuum would possess the particular velocity associated with the potentials in his theory. The concept of introducing a charge into a vacuum, without fields destroying the vacuum character of the region, is given physical reality in a theory of continual creation.

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[1] Dirac, P. A. M., *Nature*, **168**, 906 (1951).
[2] Dirac, P. A. M., *Proc. Roy. Soc.*, A, **209**, 291 (1951).
[3] Bondi, H., and Gold, T., *Mon. Not. Roy. Astro. Soc.*, **108**, 252 (1948).

A FEW remarks may clarify the relationship of the æther velocities in Bondi and Gold's theory and in mine. Where matter exists, both theories require it to have the æther velocity. Where there is no matter, Bondi and Gold interpret the æther velocity as the velocity of the matter which gets created by their process of continual creation. I interpret it as the velocity which a small electric charge would have if it were introduced. Now an electric charge cannot be introduced without violating the law of conservation of electricity, so one may wonder whether this velocity has any meaning.

Modern dynamical theory is founded on variation principles. A variation principle requires one to make a small change in the physical conditions, thereby violating some of the laws of Nature, and studies its effect on the equations of motion. *The more powerful the variation principle, the greater the number of laws of Nature which are considered to be violated.*

With my new theory some of the violations involve the creation of small charges. The theory does not allow the velocity of these charges to be arbitrary, but makes it quite definite, and so provides an æther velocity.

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Dirac's New Electrodynamics

THE content of Dirac's[1,2] beautiful new classical theory of electrodynamics can be clarified by consideration of the energy and momentum tensor.

In Dirac's gauge, the potentials A^{μ} satisfy

$$A^{\mu}A_{\mu} = k^2 \quad \text{or} \quad A_0^2 = k^2 + \mathbf{A}^2 ;$$

and the Lagrangian

$$\mathcal{L} = -\tfrac{1}{4}\int F_{\mu\nu}F^{\mu\nu}\,d^3x = \tfrac{1}{2}\int (\mathbf{E}^2 - \mathbf{H}^2)\,d^3x$$

may be treated as a function of co-ordinates $\mathbf{A}$ to yield total field energy and momentum :

$$\mathcal{H} = \int \{\tfrac{1}{2}(\mathbf{E}^2 + \mathbf{H}^2) - A^0 \operatorname{div} \mathbf{E}\}\,d^3x,$$
$$\mathbf{P} = \int (\mathbf{E}\times\mathbf{H} - \mathbf{A}\operatorname{div}\mathbf{E})\,d^3x.$$

The first terms in $\mathcal{H}$ and $\mathbf{P}$ are the energy and momentum of the Maxwell field, the second represent an energy-momentum density of amount :

$$-A^{\mu}\operatorname{div}\mathbf{E} = -\rho A^{\mu} = (-\rho/e)\,mv^{\mu},$$

where ρ is defined as div $\mathbf{E}$, and $v^{\mu} = (e/m)\,A^{\mu}$. If $(m/e)^2 = k^2$, v^{μ} is Dirac's four-velocity satisfying $v^{\mu}v_{\mu} = 1$. This is the energy-momentum density of a gas of particles with charge to mass ratio $-e/m$ and velocity distribution v^{μ}, treated as a continuous fluid. In agreement with this interpretation the field equations derived from $\mathcal{L}$ are Maxwell equations with current density :

$$\mathbf{j} = A_0^{-1}\mathbf{A}\operatorname{div}\mathbf{E} \quad \text{or} \quad j^{\mu} = (\rho/v_0)\,v^{\mu}.$$

If conventionally one assumes v^0 positive, the theory may require both signs of e/m.

Solutions with vanishing fluid density have $j^{\mu} = 0$ and correspond to free electromagnetic fields which may be derived from Maxwell's equations. The streaming velocity v^{μ} still exists[2] and, as shown by Dirac[1], may be derived from any potentials $A^{\mu *}$ which yield the desired field-strengths by gauge transformation :

$$A^{\mu} = A^{\mu *} + \partial^{\mu}S.$$

S is determined by (1), which gives explicitly,

$$\frac{\partial S}{\partial t} = \{k^2 + (\mathbf{A}^* - \operatorname{grad} S)^2\} - A^{0*}.$$

The solution involves an arbitrary function, say, the value of $S(\mathbf{x},t)$ at $t = 0$; thus the initial streaming velocity v^{μ} of the 'æther' is arbitrary. This freedom was not allowed by the old æther theories[3] ; for example, Neumann assumed that the velocity of the æther was in the direction of $\mathbf{H}$, and Fresnel that it was in the direction of $\mathbf{E}$.

It is of interest to determine the velocity field in simple cases.

(1) A Coulomb field :

$$E_r = Q/r^2, \qquad \mathbf{H} = 0.$$

The simplest solution is

$$A^0 = Q\left(\frac{1}{r} + |k/Q|\right), \qquad A^r = \sqrt{(A_0^2 - k^2)},$$

or

$$v^0 = 1 + \frac{1}{r}|Q/k|, \qquad v^r = \sqrt{(v_0^2 - 1)} ;$$

but the equations are not satisfied at the origin where the charge is concentrated.

(2) A free plane wave :
Consider

$$E_x = \omega a \cos\omega\,(z-t) = H_y,\; E_y = E_z = H_x = H_z = 0,$$

of which the simplest representation is

$$A_x^* = a \sin\omega\,(z-t).$$