

was well stressed in the House of Lords debate. Finally, it would help in the linking up of the life of the Civil servant with some of the wide issues of the national life to which Sir Edward Bridges referred as providing one of the abiding satisfactions that offset the disappointments and frustrations inevitable in such a profession.

RADIATIVE TRANSFER IN TERMS OF INTEGRAL EQUATIONS

Radiative Transfer

By Prof. S. Chandrasekhar. (International Series of Monographs on Physics.) Pp. xiv+394. (Oxford: Clarendon Press; London: Oxford University Press, 1950.) 35s. net.

THE purpose of this book is, according to the preface, to present the subject of radiative equilibrium in plane-parallel atmospheres as a branch of mathematical physics, according to a new method involving the use of non-linear integral equations.

Integral equations have had a somewhat chequered career in mathematical physics. At first the hope was raised that the use of such equations might mean a great strategical advantage in the attack on various formidable mathematical problems presented by Nature. The theory of tides may be a good example (Poincaré), and most of the simpler oscillation problems of hydrodynamics are in the same category. Thus far, however, the differential equations seem to have held their position intact. But, within the special region of radiative transfer problems, the integral equations do provide a competitive way of approach, though even here it may be premature to proclaim the final victory.

It seems that integral equations were first introduced into radiation problems by Hilbert (1912) in a novel proof of Kirchhoff's law. Hilbert even maintained that the integral equations provided the only possible method of proving this law. The following year, L. V. King, in a paper on the Rayleigh scattering of sunlight in the terrestrial atmosphere, showed how the problem could be reduced to the solution of an ordinary linear integral equation of the Fredholm type. Another year after that (1914), K. Schwarzschild showed that the problem of the radiative equilibrium of an atmosphere leads to a Fredholm equation, and Milne, ten years later, extended Schwarzschild's work by showing how different transfer problems all lead to similar types of integral equations. So the matter grew step by step, and in 1934 E. Hopf collected and condensed the available contributions into a neat little book, exceedingly well written, and published as Cambridge Mathematical Tract No. 31.

The present book, which is five times the size of that of Hopf, carries the matter further in various ways. First of all, several introductory chapters are devoted to a careful definition of fundamental terms. The equation of transfer is derived for various representative cases like that of a purely scattering atmosphere with a conservative or non-conservative flux. Diffuse reflexion is considered, and it is shown how to handle problems in which the polarization of the light beam is important. The scattering power of the atmosphere may be either isotropic or anisotropic according to some given law, for example, that of Rayleigh.

Having in this way, so to speak, cleared the deck for action, the next chapters (4-8) are devoted to the development of the method of obtaining solutions by the use of non-linear integral equations. The lack of linearity does not seem to introduce any complications, as the equations may be handled essentially in the Fredholm way by replacing the integrals by a set of ordinary linear equations. The next solution is obtained by passing to the limit of an infinite number of equations, but it appears that in practice one may go a long way with quite a small number of steps. In fact, the original procedure of Schuster of replacing the radiation field by an outgoing and an ingoing stream is, in a certain sense, equivalent to replacing an integral equation by two ordinary linear equations.

Much emphasis is placed on certain novel principles of invariance, which were first formulated by the Soviet astronomer Ambarzumian. Chapter 4 is largely devoted to the discussion of these principles, to which the author concedes such great importance that he says in the preface that "the employment of certain general principles of invariance . . . has been my justification for writing this book". It seems, however, that these principles do not replace completely the physical considerations usually made in the theory of radiative transfer. In some cases, appeal must be made to supplementary considerations in order to make the solution unique (see §§ 61 and 62.2).

The subsequent chapters (9-12) are all in the nature of applications of the general theory as developed in preceding chapters. Chapter 9 is concerned with the problem of diffuse reflexion and transmission, Chapter 10 with planetary atmospheres, and Chapters 11 and 12 contain applications to stellar problems.

The book was planned as a mathematical work on non-linear integral equations, and the author has faithfully stuck to his programme. The inclusion of the two chapters on stellar problems seems to embarrass the author slightly. In fact, in the preface it is suggested that his view was biased in this matter by his astrophysical profession. Considering the fact that the majority of prospective buyers and readers of this book will be astrophysicists, this would seem to be carrying adherence to principles too far.

It will be interesting to watch further developments and to see how far this book will blaze a new trail for the use of integral equations in theoretical physics. Perhaps the change-over from linear to non-linear integral equations, by Laplace transforms or otherwise, may provide a practical way of handling oscillation problems hitherto considered intractable. The fact that the present book establishes a temporary ascendancy of integral equations in radiation theory should act as a strong encouragement for investigations along the same line in other fields. On the other hand, considering the still rather forbidding mathematical formalism required for the presentation of radiation theory in the new way, proponents of other methods of approach have no reason for being discouraged as yet.

The book is written with great care and should not be difficult for the reader, provided he has some taste for mathematical formalism. Inherently abstruse mathematics is generally avoided. The printing is very well done, and misprints are scarce. The author is to be heartily congratulated for having produced a work of high scientific standing, which at the same time has a distinctly personal flavour.

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