Magnetism and the Rotation of Celestial Bodies

In a recent letter, H. P. Berlage[1] has ingeniously attempted to use the idea of a radial separation of electric charge, associated with degenerate matter in the interior of rotating celestial bodies, in order to derive Blackett's formula[2], which seeks to connect the angular momentum of such a body with its magnetic moment.

One can show that, in order to account for the main magnetic field of the *earth*, Berlage's model requires that some 10^6 or more electrons/cm.^{3}, from an appreciable part of the earth's core, would need an energy of the order of 10^{17} volts, in order to produce the necessary migration and separation of charge; as a result, there would have to exist an electric field of 10^9 or more volts/cm. at the boundary of the core.

Since the earth's core is rather widely thought to be metallic, for one reason or another, it is appropriate to examine the possibility that an appreciable separation of electric charge may occur in a conductor subjected to the high pressure and temperature of the earth's deep interior.

Pressure acting alone could cause a charge separation of the proper sign; but, as has recently been pointed out[3], the effect to be expected is very small. Temperature, acting with pressure, does not seem to help appreciably, as can be seen by considering the Fermi–Dirac distribution of the conducting electrons in a compressed conductor at high temperature. The number of electrons per cm.^{3} per unit of energy may be written

$$\rho_E = \frac{\nu\sqrt{E}}{1 + \exp\left(\frac{E - E_m}{kT}\right)},$$

where E is the energy of the conducting electrons, $\nu = \frac{4\pi}{h^3}(2m)^{3/2}$ and, closely enough, $E_m = \frac{h^2}{2m}\left(\frac{3n}{8\pi}\right)^{2/3}$, in which h is Planck's constant, m is the effective mass of the electron, T is the absolute temperature, k is Boltzmann's constant and n is the number of conducting electrons per cubic centimetre. E_m is the maximum energy of the conducting electrons at 0° K.

First consider the effect of compression alone. Taking convenient round numbers, assume E_m to be 10 volts before compression, and that the pressure deep in the earth increases the density there by a factor of 8. (This is an assumption favourable to the theory; actually, it is unlikely that so large a compression could occur.) If m and n remain constant during compression, the maximum energy of the electrons after compression, E_m', is 40 volts. We are probably not justified in treating m and n as constants during so great a compression; but this assumption cannot introduce an error large enough to alter the conclusions that follow below.

Secondly, let us take into account the effect of temperature, assuming T to be 10,000° K. (The temperature deep inside the earth is quite uncertain, but a rough value will do for these calculations; 10,000° K. is not unreasonable.) On carrying this out, we find that ρ_E is a maximum when E is about 36 electron volts. Owing to the relatively small effect of temperature, ρ_E decreases rapidly with increasing E. As a result, we find, for example, that for $E = 85$ volts, ρ_E is about 1 electron per cm.^{3} per electron volt. In other words, there is only about one electron per cubic centimetre having an energy as high as 85 volts, where we should like to find some 10^6 or more electrons having energies of the order of 10^{17} volts.

Thus, for an earth with a conducting core, the difference between the predictions based on the Fermi–Dirac statistics, and the requirements of the charge-separation theory, are so great that it is hard to see how terrestrial magnetism can be due to any appreciable extent to this effect.

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[1] Berlage, H. P., *Nature*, **165**, 242 (1950).
[2] Blackett, P. M. S., *Nature*, **159**, 658 (1947).
[3] Benfield, A. E., *Phys. Rev.*, **75**, 211 (1949).

Origin of Sunspots

Although astronomers have commonly believed that sunspots are deep-seated vortices, the chief characteristics of a spot (an intense magnetic field and a cool umbra) have never been successfully explained.

I wish to suggest a new theory of spot origin. One basic assumption is that the sun possesses a general magnetic field, the polar intensity of which is probably only several gauss, from ten or twenty times less than the value usually quoted. The form of the solar corona, which strongly suggests the lines of force emanating from a magnetized sphere, justifies the assumption.

If an ionized and highly conducting gas moves with velocity v in the presence of a magnetic field H, an electric field E results:

$$E = \frac{1}{c} v \times H. \quad (1)$$

Taking the curl of this equation, and employing the Maxwell equations, we find that

$$\frac{dH}{dt} = H \cdot \nabla v - H\nabla \cdot v. \quad (2)$$

Introducing the equation of continuity, we finally get:

$$\frac{d}{dt}\frac{H}{\rho} = \frac{H}{\rho} \cdot \nabla v, \quad (3)$$

where ρ is the matter density of the ionized gas. Equations (2) and (3) show that the lines of force move with the material and that changes of gas intensity generally imply changes in intensity of magnetic field.

Clouds of ionized gas, ejected from the solar poles in 'spicule activity', can cascade back to the solar surface near the equator. The influx of gases can produce, in accord with (3), a marked intensification of the magnetic field.

An examination of generalized hydrodynamic equations shows that the ionized gas will collapse when the gravitational acceleration exceeds the buoyancy effect of the magnetic field. The general characteristics of many varieties of prominences can thus be explained. Sunspots may result from complete collapse of the field down to the solar surface. The downfalling material will generally flow out away