Application of the Kozeny Equation to Consolidated Porous Media

RECENTLY, Carman[1] discussed the applicability of the Kozeny equation to the determination of the specific surface area of consolidated porous media and concluded that it should be applicable if certain specified conditions were fulfilled. Carman apparently envisaged the use of the Kozeny equation with a constant, k, of 5·0, which is the constant found applicable to the case of fluid flow through unconsolidated porous media. Carman has shown[2] that k may be expressed as the product of two parameters, namely, a shape factor, k_0, and a term $(Le/L)^2$ which we shall call tortuosity, T. He has adduced experimental and theoretical evidence to show that the variation of k_0 with pore geometry is likely to be within the approximate range 2·0–2·5 for most porous media, and this conclusion appears to have received general confirmation. For unconsolidated porous media in the porosity range 0·25–0·90, Carman has suggested that k_0 is about 2·5, and it is thus implicit that within this range T has a rather constant value of $5 \cdot 0/2 \cdot 5 = 2 \cdot 0$.

Table 1

ε	F (Carman)	F (Fricke and Morse)	F (Archie)
0·25	5·7	5·5	6·1
0·30	4·7	4·5	4·8
0·35	4·0	3·8	3·9
0·40	3·5	3·3	3·3
0·45	3·1	2·8	2·8

We suggest that T can be derived from the ratio of the resistivity of a porous medium saturated with a conducting fluid to the resistivity of the fluid. This ratio has been shown by Archie[3] to be a constant for any porous medium and was given the name formation factor, F, by him. Archie showed empirically that for unconsolidated porous media $F = \varepsilon^{-1 \cdot 3}$. We have shown that $F = T^{1/2}/\varepsilon$, and details of the derivation are being published elsewhere.

Table 1 gives a comparison of values of F computed from $(2 \cdot 0)^{1/2}/\varepsilon$ (Carman), $(3 - \varepsilon)/2\varepsilon$ (Fricke and Morse[4] for dispersed non-conducting spheres) and $\varepsilon^{-1 \cdot 3}$ (Archie).

It will be observed that the comparison is reasonably good. One reason for the discrepancy between Archie's values of F and those computed from the Carman tortuosity seems certainly to lie in the fact that no allowance for a decrease in tortuosity with increase in porosity is made in the latter formulation; that is, we believe that the relative constancy of k observed by Carman in the case of unconsolidated porous media is the fortuitous consequence of compensating changes in k_0 and T with change in ε. However, if k_0 is relatively constant over a wide range of geometrical pore shapes—and the recent results of Coulson[5] support this view—k can be calculated if T can be separately determined. If T is derived from an electrical measurement and k_0 assumed, a value of k which may greatly exceed 5·0 may be computed. This k is then used in the Kozeny equation, $S_p = (\varepsilon/kK)^{1/2}$, where K is the coefficient of permeability, to compute S_p, specific surface per unit of pore volume. The accuracy of such computations is difficult to check since there is no suitable independent method for measuring the specific surface area of consolidated media; the gas adsorption method does not necessarily measure the same surface area as that determined in flow experiments. Carman has himself shown[6], however, that in unconsolidated porous media the expression P_D/σ appears to give a close approximation to the specific surface area per unit of pore volume. P_D is the pressure that must be applied to a non-wetting phase in order to initiate displacement of a wetting fluid from a saturated porous medium, and σ is the interfacial tension between the two phases.

We have investigated the agreement between surface areas of consolidated porous media based on values of $k = \varepsilon^2 F^2 k_0$ with those computed from the relationship P_D/σ.

Table 2 shows a few results of measurements we have made, and values of S_p which have been calculated from them. A value of $k_0 = 2 \cdot 5$ was assumed. The measurement of P_D can only be made with accuracy if the pore-size distribution of the media used is reasonably uniform. To ensure this, measurements have been made on consolidated porous media fabricated by sintering 'Pyrex' and alundum grains of rather uniform particle sizes (supplied by Corning Glass Works, New York, and the Norton Company, Worcester, Massachusetts, respectively). However, measurements on certain natural consolidated sandstones, the results from two being quoted here, have also been carried out and have been found very reproducible. The fact that consolidated porous media may be prepared with smooth faces materially assists the accuracy with which an observation of P_D can be made.

It will be observed that the agreement between the surface areas determined by the two methods is generally satisfactory. The calculation of values of S_p based on values of T derived from measured values of F thus appears promising.

Table 2

Material	K (cm.^{2} × 10^{-11})	ε	F	k	P_D (dynes/cm.^{2} × 10^4)	σ (dynes/cm.)	$S_p = S_D/\sigma$ (cm.^{2}/cm.^{3})	$S_p = (2 \cdot 5\, \varepsilon\, K)^{-1/2} F^{-1}$ (cm.^{2}/cm.^{3})
Alundum	680	0·258	13·7	31·2	5·17	43·0	1,200	1,100
Alundum	660	0·254	11·3	20·5	7·24	43·0	1,680	1,370
Alundum	63	0·236	11·5	18·4	29·2	54·2	5,400	4,500
Pyrex	8·1	0·374	4·4	6·8	124·0	54·2	22,900	26,100
Pyrex	3900	0·286	6·2	7·8	5·65	54·2	1,030	870
Nicholls Buff SS	230	0·200	12·5	15·6	11·9	43·0	2,760	2,360
Berea SS	890	0·225	11·7	17·3	6·02	43·0	1,400	1,200

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