Masses of the π- and μ-Mesons

THE ability to account quantitatively for the observed masses is certainly one of the tests a successful meson theory must meet. Experimental difficulties, however, make precise mass determinations almost impossible; and while a few theoretical attempts have yielded discrete mass values[1], the experimental results are so indefinite as to make most comparisons inconclusive. More than fifty different measured masses, ranging from a few times that of the electron to around 25,000, have been reported by various investigators. In fact, there is some doubt as to whether the masses have unique values.

However, there are two values that are found repeatedly and that lend themselves to a more precise determination. These are the masses of π- and μ-mesons. The best present experimental determinations seem to be 276 and 210 respectively[2]. Their ratio, which is more reliable than the individual mass values, is stated as $1 \cdot 32 \pm 0 \cdot 01$.

Since a satisfactory theoretical prediction of these numbers has not yet been given, it was decided to examine the question from what might be called a pragmatic point of view, in the hope that numerical relations disclosed in this way might suggest the basis for a more definitive theoretical treatment. It was felt that the question of the masses must be bound up in some way with fundamental constants such as $hc/2\pi e^2$ ($= 137$) and with the properties of space. One notices, first, that the numerical difference of m_π and m_μ is very nearly 137/2 electron masses. Moreover, the formal analogy that appears to hold between the behaviour of photons and electrons on one hand and mesons and nucleons on the other suggests the existence of a 'quantum' of meson mass, corresponding to the quantization of photon energy. This quantum of mass may well be

$$\Delta m = m_\pi - m_\mu = \frac{137}{2} \text{ electron masses.} \qquad (1)$$

(In what follows, all masses will be written in terms of the electron mass.)

In addition, it is noted that, with the above numerical values, we can write with very good approximation

$$m_\pi m_\mu = \pi(137)^2. \qquad (2)$$

This relation suggests that Δm is, apart from a factor of order unity, the geometric mean of the masses of the two principal mesons. Combining (1) and (2) leads to a quadratic equation determining m_μ:

$$m_\mu^2 + \frac{137}{2} m_\mu - \pi(137)^2 = 0, \qquad (3)$$

the positive root of which is

$$m_\mu = \frac{137}{4}\left(\sqrt{16\pi + 1} - 1\right). \qquad (4)$$

Also, m_π is given by an equation differing from (3) only in the algebraic sign of the second term. Its positive root is

$$m_\pi = \frac{137}{4}\left(\sqrt{16\pi + 1} + 1\right). \qquad (5)$$

Reducing (4) and (5) to numerical values,

$$m_\mu = 211 \cdot 0, \; m_\pi = 279 \cdot 5; \qquad (6)$$

and the ratio of the two masses turns out to be

$$\frac{m_\pi}{m_\mu} = 1 \cdot 325, \qquad (7)$$

in excellent agreement with present experimental data.

One notices further that the proton mass is given by

$$m_p = 2\pi(\pi - 1)(137). \qquad (8)$$

The numerical value of this expression is $1843 \cdot 5$, which differs from the best averaged experimental value[3] of $1836 \cdot 6$ by less than four-tenths of one per cent.

Comparison with (4) and (5) brings out the interesting and suggestive relations

$$\frac{1}{m_\mu} - \frac{1}{m_\pi} = \frac{\pi - 1}{m_p}, \qquad (9)$$

and

$$m_\mu^2 + m_\pi^2 = \frac{m_\pi^2}{2\pi(\pi - 1)^2}. \qquad (10)$$

The possibility exists that all meson masses differ from m_μ (or m_π) by integral multiples of $\Delta m = 137/2$. It would be premature, however, to compare with experimental values until the latter are more accurately known.

It is worthy of note, however, that the smallest positive mass value that can be computed on the Δm basis, namely, $m_\mu - 3\Delta m$, amounts to $5 \cdot 5$. This may be one of the very light mesons for the existence of which Auger and more recently Cowan have found some experimental evidence.

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Fine Structure of the Rayleigh Line in Amorphous Substances

As is well known, when monochromatic light scattered by a liquid is examined under high resolution it exhibits a fine structure: an undisplaced central line and two lines on either side with wavelengths slightly different from that of the incident light. The appearance of the displaced components was first predicted by Brillouin[1]. On the basis of his theory, the observed displacements of frequency are regarded as a Doppler effect arising from the reflexion of the light wave by the progressive sound waves of thermal origin in the scattering medium. The frequency shift of the so-called Brillouin components is given by the formula

$$d\nu = \pm 2\nu \frac{v}{c} \sin \frac{\theta}{2},$$

where v and c are the velocities of sound and light in the medium and θ is the angle of scattering. That the effect contemplated by Brillouin does arise in liquids and crystals is now a well-established experimental fact.

Both Gross[2] and Venkateswaran[3], who made detailed studies of the fine structure of the Rayleigh