a particular magnet, the mean of the 'erect' and 'inverted' readings will still determine the true direction of the magnetic meridian.

As I have failed to find any reference to this effect in the literature immediately available to me, and as other considerations (such as the design of declination magnets, the effect of reversing the polarity of a dip needle, etc.) are involved, I should be interested to learn whether this effect has been previously reported, or encountered, by other observers.

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Figure of Merit for Servo Motor

A SERVO motor must control the position of a mechanism (valve, rudder, gear, etc.) in accordance with varying requirements. The figure of merit of such a motor indicates how speedily the motor is able to respond to these requirements.

A careful search of the literature shows that previous authors give as the figure of merit the torque to inertia ratio of the motor; H. Harriss takes exception to this in a recent paper by indicating that this figure does not give correct results¹. The torque to inertia ratio is evidently an exact measure of the motor acceleration without load. In cases in which a load is connected to the motor through a reduction gear, the motor acceleration is

$$\ddot{\phi}_M = \frac{T}{I_0 + \frac{1}{n^2} I_L}, \quad (1)$$

where $\ddot{\phi}_M$ is motor acceleration, T is motor torque, I_0 is motor armature inertia, n is gear ratio, I_L is load inertia. Therefore, the torque to inertia is still the figure of merit for maximum motor acceleration in most practical cases where $I_L/n^2 \ll I_0$. It is easy to believe that this ratio also implies maximum load acceleration, unless it is remembered that we may choose the gear ratio. The optimum gear ratio distributes the kinetic energy equally between motor and load, as will be shown herewith.

$$\ddot{\phi}_L = \frac{nT}{n^2 I_0 + I_L}, \quad (2)$$

where $\ddot{\phi}_L$ is the load acceleration. $\ddot{\phi}_L$ must be maximum or the reciprocal a minimum. Therefore,

$$\frac{\partial \left(\frac{1}{\ddot{\phi}_L} \right)}{\partial n} = \frac{I_0}{T} - \frac{1}{n^2} \frac{I_L}{T} = 0,$$

from which follows $n = \sqrt{I_L/I_0}$.

This value of n inserted into equation (2) yields

$$\ddot{\phi}_L = \left(\frac{1}{2\sqrt{I_L}} \right) \left(\frac{T}{\sqrt{I_0}} \right) \text{ (must be maximum).}$$

For any given load I_L the figure of merit is

$$T/\sqrt{I_0}.$$

As an example, let us compare two motors.

Motor No. 1	$T = 2$ oz.-in.	$I_0 = 0.03$ oz.-in. ²
Motor No. 2	$T = 8$ oz.-in.	$I_0 = 0.3$ oz.-in. ²

Formula for maximum load acceleration:

$T/\sqrt{I_0} =$	Motor No. 1 11.6	Motor No. 2 14.4
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Of the two motors, motor No. 2 is the better.

Using the formula T/I_0 would indicate incorrectly that motor No. 1 is the better.

	Motor No. 1	Motor No. 2
$T/I_0 =$	66.7	26.7

The new figure of merit is valid in all cases where friction and gear inertia can be neglected.

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¹ Amer. Inst. Elect. Eng., Tech. Paper No. 47-12 (November 1946).

Recurrence Times

CHANDRASEKHAR has remarked¹ that "the theory of density fluctuations as developed by Smoluchowski represents one of the most outstanding achievements in molecular physics", and has an important bearing on the interpretation of the Second Law of Thermodynamics. In view of the great value of Chandrasekhar's review of this and other stochastic problems in physics and astronomy, it seems desirable to correct some somewhat subtle errors of principle in his discussion of Smoluchowski's theory² of recurrence times, in case these have not previously been noticed.

For any stationary process (that is, in statistical equilibrium) with a finite number of possible states, we may define a 'lifetime' t and a 'recurrence time' θ for any specified state ' n '. Chandrasekhar's method of deriving the complete distribution for t (chapter 3) applies to any Markoff process (that is, a process the 'memory' of which does not extend beyond the previous instant); even then it is erroneous to discuss θ by applying a similar argument to the state 'not n '. The latter is a mixed state, and a process does not remain Markovian if some of its states are not distinguished. Consider, for example, a deterministic (and hence Markoff) system defined for discrete time and with only three states A , B and C , which follow each other cyclically. Given 'not A ' at any time, the system must be in state B or C , and hence in C or A at the next instant; but given 'not A ' on two consecutive occasions, the system at the next time must be in A , and thus has a memory, as regards the states A and 'not A ', extending back before the previous instant. Actually, in the theory of density fluctuations, even the state n is a mixed state, referring to the number of particles in an element of volume which itself is a sum of smaller elements.

However, we may always obtain formulae for the average values T and Θ of t and θ (suitably defined) by general arguments applicable to any stationary process for which the ergodic property holds, and essentially identical with Smoluchowski's original arguments. For example, in a long enough total time interval I , the time spent in any state n will be IP , where P is the probability of state n . Hence the number N of excursions from n to 'not n ' is given by $NT=IP$. The total time available for such excursions before return to n is $I(1-P)$, whence $\Theta P=T(1-P)$. Smoluchowski's relation connecting T with the conditional probability of n , given n at the previous instant, also holds in general.

The definition of θ implied in the above argument is that of a true recurrence time, and is not the same as Chandrasekhar's time of recurrence to n from "an arbitrary state which is not n ". We may, in fact, following Smoluchowski's own distinction between the two definitions, denote the times θ on the two definitions by θ_1 and θ_2 respectively. Similarly, we