

With ε_{11} , ε_{22} , ε_{12} for 'strains' and r_1 , r_2 for direction cosines of $d\mathbf{R}$ with respect to $\mathbf{c}_1, \mathbf{c}_2$ gives the usual expression^{1,2},

$$\begin{aligned} \left(\frac{dR^0}{dR}\right)^2 &= 1 - 2U_{1,1}r_1^2 - 2U_{2,2}r_2^2 - 2(U_{1,2} + U_{2,1})r_1r_2 \\ &= 1 - 2\varepsilon_{11}r_1^2 - 2\varepsilon_{22}r_2^2 - 2\varepsilon_{12}r_1r_2. \end{aligned} \quad (6)$$

Since the dyadic in (5) is self-conjugate, then 'principal' directions $\mathbf{c}_X, \mathbf{c}_Y$, say, can be found⁴ to give,

$$d\mathbf{R} \cdot [(1 - 2U_{X,X})\mathbf{c}_X\mathbf{c}_X + (1 - 2U_{Y,Y})\mathbf{c}_Y\mathbf{c}_Y] \cdot d\mathbf{R} = (dR^0)^2. \quad (7)$$

This is an ellipse, since both the coefficients are positive; but its axes are not in the directions $\mathbf{c}_1, \mathbf{c}_2$ of the principal normal stresses S_1, S_2 . However, if θ the angle of rigid rotation of the element of substance is zero in (4), then $U_{1,1}$ and $U_{2,2}$ equal $D_{1,1}$ and $D_{2,2}$ respectively, and (5) becomes,

$$d\mathbf{R} \cdot [(1 - 2D_{1,1})\mathbf{c}_1\mathbf{c}_1 + (1 - 2D_{2,2})\mathbf{c}_2\mathbf{c}_2] \cdot d\mathbf{R} = (dR^0)^2. \quad (8)$$

This is precisely the ellipse that would follow from (2) considering 'straining-displacement', since $(dR^0 \text{ inst})^2$ equals $(dR^0)^2$, and its principal axes are in the directions of the principal normal stresses as required.

Thus it is seen that allowing just an infinitesimal 'rotation of the body as a whole' (in my sense³) through angle θ leads to the loss of one-one correspondence between principal normal stresses and 'strains' defined by 'spatial-displacement' gradients.

The argument is developed similarly in the three-dimensional case. If the rotation of the body as a whole is finite, then clearly the effects noted here will be accentuated. A recent note⁶ pointed out that the classical finite 'strains' expressions non-linear in displacement gradients are unnecessary even when 'straining-displacement' is used in their definition. If in the definition of those non-linear expressions 'spatial-displacement' be used, as is done by others than myself, then apart from their other disadvantages, such strain expressions cannot have one-one correspondence with stress, as has been shown in the present communication.

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¹ Love, A. E. H., "Mathematical Theory of Elasticity" (Camb. Univ. Press, 1934).

² Sokolnikoff, I. S., and Specht, R. D., "Mathematical Theory of Elasticity", 66, 67 (McGraw-Hill, 1946).

³ Swainger, K. H., *Phil. Mag.*, **38**, 422 (1947).

⁴ Weatherburn, C. E., "Advanced Vector Analysis" (Bell, 1937).

⁵ Swainger, K. H., *Nature*, **163**, 23 (1949).

Combustion of Liquid Fuels

A RECENT letter by Godsavé¹ described experiments on the combustion-rates of single liquid-fuel droplets. The general case of the combustion of liquid fuel in gas streams is being studied at this Laboratory, and it may be interesting to outline the approach made and the preliminary results, which will be published elsewhere².

Heat transfer between a body and a fluid flowing past it is often pictured as occurring by thermal conduction across a stagnant film of fluid adhering

to the body, while the temperature throughout the bulk of the liquid is supposed substantially constant; an analogous simplification is commonly made when matter transfer takes place in like circumstances. By assuming the thickness of the film to depend only on the nature of the fluid and the pattern of flow past the body, it is possible to predict heat-transfer coefficients from matter-transfer experiments, and vice versa.

In the present theory of liquid-fuel burning, the combustion is supposed to occur within a similar stagnant film surrounding the liquid mass. The film is divided into two parts by a surface, parallel to the film boundaries, at which the chemical reaction occurs. Fuel vapour and oxygen diffuse towards this surface from the inner and outer boundaries respectively, and combustion products diffuse outwards. A small proportion of the heat of reaction is conducted inwards to provide the latent heat of vaporization of the fuel and the enthalpy increase of its vapour, while the remainder is conducted outwards towards the gas stream. For a given fuel and atmosphere, the temperature and matter distributions can be calculated from the ordinary laws of thermal conduction and molecular diffusion; these distributions are independent of the film thickness. In particular, the temperature at the reaction surface can be found; it is different in general from the adiabatic flame temperature.

The combustion-rate can then be calculated if the thickness of the effective stagnant film is known. This thickness has been determined implicitly by many investigators in heat- and matter-transfer experiments on various systems, for example, spheres in gas streams.

It is convenient in practice to imagine the burning fuel surface to be replaced by a solid surface at such a temperature that the rate of heat addition to the gas stream is unchanged. The required temperature is calculable from the properties of the fuel and the gas, and is independent of the thickness of the stagnant film, that is, of the shape of the body and the velocity of the gas stream. Thus a kerosene surface burning in air at room temperature adds heat to the atmosphere at the same rate as would an identical solid surface at a temperature of 25,000° C. above its surroundings.

The theory has been confirmed experimentally by measuring the rate of burning of kerosene by natural convection from a spherical surface of 1½ in. diameter, the effect of latent heat variations being simulated by cooling the sphere internally. The measured rates were about 50 per cent of the theoretical, and lay on similar curves. It was found that the combustion-rate increases as heat of combustion and atmospheric temperature and oxygen concentration increase, and as latent heat of vaporization decreases. The 'volatility' of the fuel is irrelevant, when the steady state has been reached. The extinction limits of the flame and the occurrence of smoke appeared to depend on the peak temperature within the film, as did also the reaction mechanism.

Further study is being directed to combustion by forced convection.

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¹ *Nature*, **161**, 708 (1949).

² *Fuel*, **29**, Nos. 1 and 2 (1949).