are infinite. Since the electron has spin $\frac{1}{2}$, field-independent moments other than a magnetic moment cannot exist; but polarizabilities and higher induced moments exist. To calculate the magnetic polarizability it is necessary only to carry the expansion in H one step further. This we have done, and it is found that this term is convergent for large values of the momentum of the virtual photons but diverges logarithmically for low values. This divergence, however, is due to an unpermissible expansion in powers of H , as can be seen by summing up the divergent contribution of this term with other contributions occurring in the higher powers of H . The result is a term involving $H^2 \log H$. Thus we find for the contributions to the self-energy due to the static and dynamic interactions with the virtual photons:

$$\left. \begin{aligned} E_{\text{static}} &= -\frac{e^4 H^2}{90\pi m^3} + O(H^3) \\ E_{\text{dynamic}} &= -\frac{e^4 H^2}{4\pi m^3} - \frac{2e^4 H^2}{3\pi m^3} \log \frac{2eH}{m^2} + O(H^3) \end{aligned} \right\} \quad (2)$$

This result is finite, and it is quite easy to see that the terms proportional to higher powers of H are also finite. Owing to the logarithmic dependence on H , strictly speaking it is not correct to call (2) the 'magnetic polarizability', but this is of little importance. (2) is still smaller than (1) by a factor of the order $\frac{\mu_0 H}{mc^2} \log \frac{mc^2}{\mu_0 H}$ (where μ_0 is the Bohr magneton), which makes the polarizability for all practical purposes far too small to be observed at present.

I wish to express my thanks to Prof. Heitler for suggesting this investigation, and for valuable discussions.

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¹ Schwinger, *Phys. Rev.*, **73**, 415 (1948).

² Luttinger, *Phys. Rev.*, **74**, 893 (1948).

Analogue Calculating Machine for Functions of a Complex Variable

WE have recently been concerned with a physical analogue to assist in dealing with functions of the complex variable, z , of the class:

$$f(z) = \frac{a_n z^n + a_{n-1} z^{n-1} + a_{n-2} z^{n-2} \dots a_0}{b_m z^m + b_{m-1} z^{m-1} + b_{m-2} z^{m-2} \dots b_0} \\ m \sim n = 1; a_n, b_m \text{ real.} \quad (1)$$

This analogue device has many applications to electric network analysis and synthesis, and to other vibration problems; however, its capabilities would appear to be of sufficiently broad mathematical interest to warrant more general attention.

Such functions satisfy Laplace's equation $\nabla^2 f(z) = 0$, except at the singularities, and so may represent two-dimensional physical potential fields, for example, (a) an electrostatic field, (b) a stretched membrane,

(c) a current-carrying conducting sheet, etc., with the singularities represented respectively by point charges, point displacements and point current sources. The system (c) has been found particularly suitable, using a shallow layer of electrolyte and by taking certain precautions to eliminate the boundary error¹, so that the electrolyte surface represents the infinite z -plane and includes an accessible 'infinity' point. A fine point electrode, carrying a constant current, placed at the point z_r , sets up a potential at any other point z proportional to $\log |(z - z_r)|$ and a potential gradient represented in both magnitude and direction by the conjugate of $(z - z_r)^{-1}$, both of which may be measured electrically. These properties form the basis of the analogue, which is applied in two different ways.

In the first application, the real, imaginary or complex zeros of either the numerator or denominator of (1) may be found, by exploring the potential gradient over the electrolyte surface set up by suitably placed electrodes, in order to locate the points of zero gradient. Furthermore, the case of complex coefficients, $a_n, \dots b_n, \dots$ etc., may also be dealt with².

Secondly, if $f(z)$ be factorized by this method, it may be represented in logarithmic form:

$$\log f(z) = \sum_{z=1}^n \log (z - z_z) - \sum_{p=1}^m \log (z - z_p), \quad (2)$$

by electrodes placed at the zeros z_z and poles z_p carrying currents proportional to their multiplicity (if multiple singularities are present). The potential over the electrolyte surface determines the modulus of $\log f(z)$, and, further, it may be shown that its argument is represented by the summation, along a line parallel to the imaginary axis, of the potential gradients (parallel to the real axis) between z and the real axis. This modulus and argument measurement, especially along the imaginary axis, has been reported previously.

Thirdly, with $\log f(z)$ thus represented, the residues at the simple poles of $f(z)$, which are in general complex, may be measured. The residue v_p at a pole z_p is the value of $f(z) \cdot (z - z_p)$ at $z = z_p$; this may be measured physically by removing the electrode at z_p and measuring $\log f(z)_{z=z_p}$ as explained above. A somewhat more complicated process enables the residues at double or triple poles to be determined. Knowledge of all the residues enables the inverse Laplace transform of $f(z)$ to be constructed.

Calculations can also be carried out on functions of other types; in general, any regular function satisfies Laplace's equation and is capable of similar physical interpretation, though not always by using point electrodes only. For example, a branch cut may be simulated by a strip, made of conducting or insulating material, depending upon the type of discontinuity possessed by the function, placed in the electrolyte.

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¹ *Nature*, **161**, 845 (1948).

² Bloch, A., paper read before the 7th International Congress of Applied Mechanics, London (1948).