densations of that primordial gas which must have originated sooner or later according to Jeans' principle of gravitational instability[8]. Jeans' classical formula gives the diameter D of the condensations which will be formed in a gas of temperature T and density ρ in the form:

$$D^2 = \frac{10\pi}{9mG\rho} \cdot \frac{3}{2} kT. \tag{10}$$

Using the expressions (3) and (5), we get:

$$M = \rho D^3 = \frac{2^{31/8} 5^{7/4} \pi^{5/4} e \hbar^{5/4} \varepsilon^{5/4}}{3^{17/8} m^{15/4} c^{5/4} G^{7/4}} \tag{11}$$

(where a has been expressed through other fundamental constants).

It is interesting to notice that the time-factor cancels out in the calculation of M, so that the mass of the condensations comes out the same, independent of the epoch when they were formed. It seems, however, reasonable to assume that the effect of gravitational instability became important only when the mass-density of radiation became comparable with the density of matter, since it is hard to imagine a 'gravitational condensation of pure radiation'. Using (4) and (5), we find that $\rho_{\text{rad.}} = \rho_{\text{mat.}} = 3 \times 10^{-26}$ gm.cm.$^{-3}$ at $t = 3 \cdot 9 \times 10^{15}$ sec. $= 1 \cdot 3 \times 10^8$ years, at which point $T = 340°$ K. For this value of t we obtain:

$$D = \frac{2^{45/8} 5^{1/4} \pi^{7/4} e^3 \hbar^{3/4} \varepsilon^{15/4}}{3^{27/8} m^{29/4} c^{35/4} G^{5/4}}. \tag{12}$$

Substituting numerical values, we have:

$$\begin{aligned} M &= 5 \cdot 5 \times 10^{40} \text{ gm.} = 2 \cdot 7 \times 10^7 \text{ sun-masses} \\ D &= 1 \cdot 3 \times 10^{22} \text{ cm.} = 13{,}000 \text{ light-years,} \end{aligned} \tag{13}$$

which must represent the masses and the diameters of the original galaxies.

The above estimate of galactic masses falls short by a factor of about one hundred from the mass-values of galaxies obtained from astronomical data. But it must be remembered that the simple Jeans' formula used in these calculations does not take into account the effect of radiation pressure, and also is applicable only to the gravitational condensations in non-expanding space. The effect of additional radiation pressure (which is quite important according to the previous considerations) and the tearing force of expansion will lead to considerably larger condensation masses. The detailed study of this question will require, however, the extension of Jeans' classical arguments for the case of a mixture of gas and radiation in the expanding space. At the present stage one should be satisfied with the fact that, by such comparatively simple and rather natural considerations, *masses and sizes comparable to those of stellar galaxies can be expressed in terms of fundamental constants, and the basic quantities of nuclear physics.*

We may add that, according to the above picture, the galaxies have been originally formed in the purely gaseous form (including a certain amount of solid dust particles), which must account for the regular shapes of rotating bodies. The formation of individual stars within the galactic bodies must have taken place at a somewhat later stage, probably along the lines of the Spitzer – Whipple theories[9,10]. When stars were formed by the condensation process within the rotating gaseous mass, their tangential velocities, being equal to the original velocities of the gas-masses, were clearly not high enough to maintain them on circular Kepler orbits, so that *the newly formed stars must have been moving along elongated elliptical orbits with the points of maximum elongation in the places of their origin.* This situation must have remained essentially unchanged even when all the material of originally gaseous galaxies was used up in the formation of stars.

These considerations give a simple explanation of the otherwise mysterious fact that the elliptical galaxies and the central bodies of spirals rotate 'as solid bodies' with the linear velocities proportional to the distance from the axis. In fact, according to our picture, the maximum Doppler displacements observed at various distances from the axis correspond to the velocities of stars passing through 'aphelion' at these particular distances, and, according to the previous argument, are equal to the velocities which the gas-masses must have had in these regions prior to their condensation into the stars.

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COASTAL WAVES*

RECENT investigations by the Admiralty have shown that storm waves and the swell that leaves the storm area are composed of a mixture of wave-trains, the wave-lengths of which range from a few feet up to a maximum which depends on the greatest wind strength, and may be as much as 3,000 feet. It seems remarkable that such component wave-trains should travel independently across the ocean, and that all except the very shortest should be recognizable after travelling thousands of miles; but such close agreement with theory has been demonstrated, and the component wave-trains have been found to advance across the ocean with the theoretical velocities appropriate to their lengths.

The short waves formed at the beginning of a storm are overtaken and outdistanced by long waves formed when the wind is strongest. With the help of new wave-recording and analysing apparatus, each wave-length can be detected and its amplitude measured when it arrives at a distant coast, and the waves recorded on the coast of Cornwall are found at times to be a mixture of waves generated near the coast with swell-components from more than one North Atlantic storm, and with other swell, probably a few inches high, which was generated in a storm so far away as Cape Horn.

The visible crests and troughs are the result of the combination of trains of waves of different lengths, and since such waves travel with different velocities, the wave-pattern is continually changing. When some of the component wave-trains get into step and reinforce each other, they produce a group of typical waves with relatively high crests and deep troughs; but when they get out of step and tend to neutralize

* An account of the symposium held at the meeting of Section A (Mathematics and Physics) of the British Association on September 9, in which Brigadier R. A. Bagnold, Mr. N. F. Barber, Dr. G. E. R. Deacon and Major W. W. Williams took part.

each other, the wave-crests are low and irregular. Such interaction between component wave-trains explains the recurrence of sequences of high and low waves that is often observed; it is not necessarily the fifth, seventh or any other number of wave that is always the highest, but there is often sufficient regularity to suggest a rhythm, and to allow the coxswain of a boat to take advantage of one of the quiet periods. There is most likely to be a noticeable rhythm when a narrow range of wave-lengths is present, as in long swell from a distant storm. Although the variability of the wave-pattern on deep water and of the height of the crests that approach the beach depends on the interaction of component wave-trains, such interaction appears to be suspended when the waves reach very shallow water; here they rise into sharp crests separated by wide, flat troughs, and each crest seems to travel independently of its neighbours.

One of the most noticeable changes in waves entering shallow water is an increase in height. The reason, according to an explanation given by Lord Rayleigh in 1911, is essentially that the velocity of a wave decreases as the depth of water decreases, and the height of the wave increases since it must carry energy towards the shore at the same rate. The increase in height depends on the wave-length, and long swell may rise to as much as twice its deep-water height; it may become apparent for the first time when it enters shallow water, and such an emergence has earned for it the name 'ground-swell'.

When waves approach the shore obliquely, the ends which reach shallow soundings first travel slower than the seaward ends, and the waves swing round until they are nearly parallel to the beach. In this way waves can swing round headlands and islands, and enter bays not directly exposed to their original direction; the effect is more marked with long waves, and a long swell may round a headland that offers complete protection against shorter wind-waves. A shallow spit extending out to sea tends to focus the wave-energy; but a deep gulley pointing into shallow water causes the wave-energy to be refracted towards the sides and offers quieter water in the middle. Waves are also affected by tidal streams, and Mr. N. F. Barber argued that waves travelling at 20 knots in still water would make no progress against a tidal stream of 5 knots but would steepen and break. Waves meeting a tidal stream obliquely will be refracted, and the theory suggests that a tidal stream may protect an anchorage.

Brigadier R. A. Bagnold, dealing with the movement of sand and shingle, described model experiments in which it could be seen that long waves approaching a beach give rise to a strong drift of water along the bottom in the direction of wave travel; this movement is continued as far as the plunge-line, where the waves break, and there the water appears to rise from the bottom to form a backward drift at the surface. He remarked that we have no exact knowledge of what happens on the bed of the sea, although it is of great importance to engineers to predict the movements of sand and shingle; the necessary observations on the sea bed would be difficult and expensive, but it is quite possible to make them.

Mr. Barber put forward theoretical reasons for believing that the movements in the sea would resemble those observed in the model. Waves cause a small transport of water in their direction of travel, greatest at the surface but almost as large at the bottom if the waves are long. It is not likely to be affected to any appreciable extent by friction with the bottom, since the vorticity associated with the oscillatory movements of the waves is continually changing sign and therefore unable to diffuse far above the bottom. The seaward drift which must compensate for the shoreward transport can, on the contrary, be treated as a unidirectional flow for which the effect of the bottom drag will be greater, and its velocity will be a maximum at the surface and zero at the bottom. The resultant effect of bottom friction and viscosity on the wave-transport and the compensating seaward drift should be a seaward drift at the surface and a shoreward movement at the bottom; if the waves are short, there would be a shoreward movement at the surface and the bottom and a seaward movement in the intermediate depth. For waves 3 ft. high Mr. Barber estimated the velocity of such movements to be of the order of 5 cm. a second.

Little information is available of the water movements between the breakers and the beach. In the model experiments the movements in this part of the tank are difficult to observe because of the strong turbulence; but it was found that dye put into the water between the plunge-line and the beach showed little tendency to pass outwards across the plunge-line. One of the consequences of the termination of the forward movement at the plunge-line is that sand tends to accumulate there; it was also found that the sand does not move uniformly towards the shore but as a series of regularly spaced bars, the spacing being of the same order as the wave-length. In the sea, the formation of bars is complicated by the shifting of the breaker zone relative to the beach as the tide rises and falls, and they are observed particularly off coasts where the tidal range is small. Major Williams mentioned the research into the effect of waves on beaches that is being undertaken in the Department of Geography at Cambridge, and made special reference to widespread occurrence of bars in the Mediterranean Sea. They were a considerable hazard during landing operations, since men, or vehicles proofed for a certain depth, would find water too deep for them if they left a vessel which had grounded on a bar. The movements of such bars are being studied in relation to the waves and other factors.

Major Williams also described how the reduction in the velocity of waves entering shallow water had been used to obtain accurate information about the underwater profile of the beach. By comparing successive aerial photographs of known scale, the wave-length and velocity of waves at different distances from the shore could be measured and the beach profile calculated.

In the discussion which followed, it was remarked that observations and theory tend to discount reports of strong undertow below waves, and suggest that the resultant movement, after allowing for the backward and forward movement of the waves, is towards the shore. The evidence of a seaward movement at the surface appears, on the other hand, to grow stronger. On the coast of California in particular, it has been observed that strong outward movements may develop at certain points on the beach, presumably where the seaward drift is concentrated; the position and intensity of such 'rip-currents', as they are now called, may vary, but they tend to be associated with some irregularity in the beach or where some natural or artificial barrier extends to seaward. Together with the local tidal streams they may account for the danger of bathing in certain localities; they can sometimes be detected because foam or discoloured water is seen moving outwards through the breakers. It is reasonable to suppose that they are stronger with high waves, especially when a group of high waves is followed by a group of low waves, since the high waves will tend to build up a head of water on the beach. Little is known of the behaviour of breaking waves, but it has been assumed with some success that the wave breaks when the water-particle velocity, which depends on the height of the wave, exceeds the wave-velocity, which depends on the depth of water. Such an argument implies a close relationship between the height of a wave and the depth of water in which it will break; and it has been shown that there is a tendency for waves to break when the ratio of water-depth to wave-height is four-thirds.

Prof. J. D. Bernal gave an evening discourse on "Waves and Beaches" on September 13; all available tickets were issued before lunch-time on the first day of the meetings, and the close attention of the crowded hall showed a lively interest in the subject as well as appreciation of the inspiring lecture. After summarizing the studies made during and since the War, Prof. Bernal described their application to military purposes and the prevention of coastal erosion, and showed how a better knowledge of the transport of sand and shingle by waves would be useful in geological studies. In discussing coastal erosion and other coastal engineering problems, he remarked that no other industry spent such a small proportion of its outlay on research, or stood so much in need of accurate knowledge of the basic principles involved and of some central authority.

MULTIPLE ALLELOMORPHS IN COLOUR VISION

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EVIDENCE has been found that the main forms of defective red-green colour vision in man are multiple allelomorphs, alternative to each other and to the normal form[1]. In order to prove this hypothesis it would be necessary to show: (*a*) that the forms of red-green vision are discontinuous variations; (*b*) that they are inherited true to type; (*c*) that they segregate independently; and (*d*) possibly that they have an order of dominance. It will be seen that the evidence in favour of the hypothesis is very strong.

Discontinuous Variations

The question is very important whether or not the well-known variations of colour vision, two of which are generally called red-green blindness, are the extremes of a continuous normal curve. Rayleigh[1a] threw doubt on the continuous nature of these variations in 1881. He showed that there were red and green anomalous subjects who were characteristically different from the normal, from each other and from the red-green blind as well. This has been confirmed by Pickford[2]. In addition, Rayleigh indicated in the same paper that the two types of red-green blindness were also distinctively different, a conclusion supported by von Kries and Donders[3] and by Pickford more recently[4]; while Houstoun showed by means of his microscope test[5] that the major red-green defectives formed a small and rather irregular group separated clearly from the large group of normal subjects.

In a recent investigation upon some 900 normal and more than 140 red-green defective subjects (the latter not all found by chance), I have found convincing evidence of a statistical kind that there are four or probably five distinct types of red-green vision in addition to the normal form. This research will be published in full as soon as possible (*J. Psychol.*, in the press). The types are: deuteranope, protanope, green anomalous and red anomalous, the latter being divided into two classes, those with and those without the darkened red of the protanope. Whether the two types of red anomalous subjects are statistically distinct is difficult to decide finally on the basis of the eight cases available, but it is extremely likely.

Adequate tests, which will be published shortly, show that the anomalous subjects are not to be viewed as the intermediates between the red-green blind and the normal. To think of the green anomalous as intermediate between the normal and the 'green-blind', while the red anomalous might be intermediate between the normal and the 'red-blind', is highly tempting. There are, however, grave objections to this: (1) neither class of anomalous subjects is sufficiently variable to fill the gap it is supposed to occupy; (2) both the so-called 'red-blind' and 'green-blind' are, in fact, red-green blind; (3) though most red-green blind subjects are more defective, some are actually less defective than the anomalous, who are possibly to be identified with Houstoun's "colour-different" subjects[6]. Edridge-Green was well aware that the anomalous were not true intermediates[7]. It was only in terms of the Young–Helmholtz theory, now rapidly becoming more and more difficult to sustain[8,9], that they could be thought of as true intermediates; indeed, it has long been realized[10], as Rayleigh foretold[11], that the red-green blind cannot be divided into the 'red-blind' and 'green-blind' classes, but must be divided on the principle that while both types are red-green blind, in the protanopes or scoterythrous the red end of the spectrum is greatly darkened, and in the deuteranopes or photerythrous there is no darkening of the red. That about half of the red anomalous do not have the darkened red of the protanope is a fourth objection to treating them as intermediates.

It is inevitable that we should conclude that there are four (or probably five) types of major red-green vision defects, which are as clearly discontinuous from each other and from the normal as is usual with familiar Mendelian allelomorphs.

Inheritance True to Type

Few people have realized the implications, for the theory of inheritance of colour vision defects, of Rayleigh's pedigree[12] showing the green anomalous condition in three of his brothers-in-law. The seven siblings he tested are shown in Pedigree 1*. The three brothers who were green anomalous inherited the defect true to type, and were completely different

♂? × ♀ *H*?

♂ *g*   ♂ *g*   ♂ *g*   ♂ *N*   ♀ *H* or *N*   ♀ *H* or *N*   ♀ *H* or *N*

Pedigree 1*

* Throughout this paper *N* = normal, *d* = deuteranope, *p* = protanope, *g* = green anomalous, *r* = red anomalous, *H* = 'normal' heterozygote (see section on heterozygotes later in this article), ? = doubtful owing to inadequacy or lack of testing.