LETTERS TO THE EDITORS

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Behaviour of a Particle of Very Small Mass in a Magnetic Field

It would seem at first sight that a particle of very small mass, such as accompanies β-disintegration or electron-capture, and significant magnetic moment μ, would be very difficult to observe. According to Bethe[1], it should have a fairly high ionizing power and a correspondingly small penetration into matter. Moreover, if it already possesses a charge e, the uncertainty principle should play a part in all cases where experimental evidence on μ would be sought[2]. However, for an ultra-relativistic particle (that is, a particle with a high ratio of kinetic mass to rest mass) special considerations come into play which may modify the generally accepted conclusions.

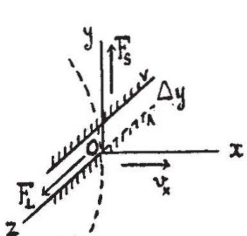
A Dirac particle can be considered as a spherule which is both magnetized and carries an electric charge. As Frenkel[3] has pointed out, if the particle has a magnetic moment $\vec{\mu}$, an electric moment $\vec{\pi}$, of the same order of absolute value, must necessarily appear. Moreover, when the speed approaches c, the three vectors $\vec{\mu}$, $\vec{\pi}$ and $\vec{c}$ tend to become orthogonal[4]. The magnitude of the moment $\vec{\mu}$ should vary with the speed if it could have any direction whatever. However, in the case where the moment $\vec{\mu}$ is orthogonal to the direction of the displacement, the former has the properties of invariance, as described by Louis de Broglie[5], who pointed out that the densities of moments $\vec{\mu}$ and $\vec{\pi}$ form the six components of an antisymmetrical tensor of the second order: the main moments that are obtained by integration from these densities have invariant values. Thus, in the relativistic case, moment $\vec{\mu}$ would always tend to follow the orthogonal direction and to be invariant in this.

Let us write the uncertainty relation in a new form that is better adapted to high speeds:

$$\Delta x \,.\, \Delta\eta \geqslant \frac{h}{m_0 c}\,, \text{ where } \Delta x \,.\, \Delta v \geqslant \frac{h}{m_0 \eta^3} \qquad (1)$$

$$(\text{for } \Delta p = m_0 c \,.\, \Delta\eta = m_0 \eta^3 \, \Delta v).$$

In the diagram, particles (e,μ), with a speed v_x in the direction Ox, cross a long slit in the Z direction which has the width Δy.



The Lorentz force at the origin O is $F_L = ev_x H(O)$. The width Δy of the slit introduces in this force an uncertainty

$$\Delta F_L = e\left[v_x\left(\frac{\partial H}{\partial y}\right)\Delta y + H(O)\Delta v_x\right],$$

which we must compare to the force F_S due to spin. The ratio of these forces, with $\mu = eh/4\pi m_0$, will be

$$\frac{\Delta F_L}{F_S} = \frac{4\pi m_0}{h}\left[v_x \Delta y + \frac{H}{\partial H/\partial y}\,\Delta v_x\right], \qquad (2)$$

putting, from (1),

$$\Delta y \geqslant \frac{h}{m_0 \eta^3 \Delta v_y} \text{ and } \Delta v_x \geqslant \frac{h}{m_0 \eta^3 \Delta x}. \qquad (3)$$

(2) then becomes

$$\frac{\Delta F_L}{F_S} \geqslant \frac{4\pi m_0}{h}\left[\frac{h}{m_0\eta^3}\cdot\frac{v_x}{\Delta v_y} + \frac{h}{m_0\eta^3}\,\frac{H}{\partial H/\partial y}\,\frac{1}{\Delta x}\right], \text{ or}$$

$$\frac{\Delta F_L}{F_S} \geqslant \frac{4\pi}{\eta^3}\left[\frac{v_x}{\Delta v_y} + \frac{H}{\partial H/\partial y}\,\frac{1}{\Delta x}\right].$$

The term in brackets being obviously greater than 1, we finally get

$$\Delta F_L / F_S \gg 4\pi/\eta^3, \qquad (4)$$

and the magnetic moment μ will be observable if

$$\eta^3 \gg 4\pi, \qquad (5)$$

a condition that is obviously fulfilled for every relativistic particle.

The magnetic moment of a charged light particle might thus be found.

Even if we deny invariance to the magnetic moment and allow that its absolute value might decrease as the kinetic mass of the particle increases, the various expressions which have been given[6] for the conditions of observability remain, with a power of η diminished by one unit, and the conclusions are unchanged.

The foregoing considerations are of a very general character. The question will be examined afresh with the Dirac method. On the other hand, I shall shortly show that a relativistic particle may give a weaker ionization when traversing matter than was previously thought. This arises both because of the orthogonal relation of the electric moment $\vec{\pi}$ to $\vec{\mu}$ and $\vec{c}$, and because of the contracting of the magnetic and electric lines of force in a plane normal to the direction of its displacement.

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May 8.

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[3] Frenkel, *Z. Phys.*, **37**, 4–5, 243.
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Feret's Statistical Diameter as a Measure of Particle Size

Various measures of the size of irregularly shaped particles as seen in profile under the microscope have been used, chosen according to their theoretical significance or practical ease of measurement. These include, using Heywood's notation[1,2]: (i) the diameter of the circle of equal area, d; (ii) the diameter of the circle of equal perimeter, D; (iii) the length of line bisecting the profile area (Martin's statistical diameter[3]), M; and (iv) the perpendicular distance between parallel tangents touching opposite sides of the profile (Feret's statistical diameter[4]), F. M and F