authors using high-voltage arcs, ozonizers and glow discharges all support this result.

These investigations were undertaken in connexion with work in progress here on the spectrum of the night sky, and from the spectra measurements are being made which will enable the O_2 Herzberg bands, believed present in the night sky spectrum, to be more accurately predicted than previously.

I am indebted to Prof. R. W. B. Pearse for suggesting the work and for his constant advice.

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[1] Lalji Lal, *Nature*, **161**, 477 (1948).
[2] Wulf, O. R., and Melvin, E. H., *Phys. Rev.*, **55**, 687 (1939).
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A Simple Result in Quadrature

In calculating the intensity of light scattered by a transparent homogeneous medium, one comes across the infinite series[1]

$$\alpha \sum_{n=-\infty}^{+\infty} \frac{\sin^2 (n\alpha + \theta)}{(n\alpha + \theta)^2}, \qquad (1)$$

in which θ is a constant and n an integer. α is a positive number, which under the conditions under which light-scattering is generally studied is very small, and hence the sum of the above series is usually replaced by the corresponding integral

$$\int_{-\infty}^{+\infty} \frac{\sin^2 x}{x^2}\, dx, \qquad (2)$$

which evidently is equal to π.

It can be shown[1], however, that the equality of (1) and (2) holds not only in the limit when $\alpha \to 0$, but for any value of α in the range $0 < \alpha \leqslant \pi$. An obvious interpretation of this result is that the area subtended between the curve $y = \sin^2 x/x^2$ and the x-axis can be obtained by taking the sum of the ordinates at equal intervals α, $0 < \alpha \leqslant \pi$, and multiplying by α, that is, by simple rectangulation, just as well as by integration. In erecting these ordinates at equal intervals, we may start from any value θ of x.

The main purpose of this note is to direct attention to this property, namely,

$$\alpha \sum_{n=-\infty}^{+\infty} f(n\alpha + \theta) = \alpha \sum_{n=-\infty}^{+\infty} f(n\alpha) = \int_{-\infty}^{+\infty} f(x)\, dx, \qquad (3)$$

where $0 < \alpha \leqslant$ a certain constant α_0, which characterizes $f(x) = \sin^2 x/x^2$, and to show that numerous other functions can be constructed that have this property. A method of evaluating (1) given by Prof. Wiener, and quoted in the paper referred to[1], suggests the criterion by which to construct such functions.

Consider an even function $F(x)$ $[= F(-x)]$ the Fourier transform of which,

$$g(v) = \frac{1}{\sqrt{(2\pi)}} \int_{-\infty}^{+\infty} F(x) \exp (ivx) dx, \qquad (4)$$

has non-zero values when $|v|$ is less than a certain constant v_0, and is zero otherwise. Then by Poisson's summation formula[2]

$$\alpha \sum_{n=-\infty}^{+\infty} F(n\alpha + \theta) = \sqrt{(2\pi)} \sum_{N=-\infty}^{+\infty} \exp [-i2\pi N\theta/\alpha] . g(2\pi N/\alpha), \qquad (5)$$

where n and N are integers and $\alpha > 0$. If now α is chosen to be $\leqslant 2\pi/v_0$, then all the terms on the right-hand side of (5) except the one corresponding to $N = 0$ vanish, and we obtain

$$\alpha \sum_{n=-\infty}^{+\infty} F(n\alpha + \theta) = \sqrt{(2\pi)}\, g(0),$$

which can be seen from (4) to be equal to $\int_{-\infty}^{+\infty} F(x)\, dx$; and hence the function F satisfies (3).

Since the Fourier integrals of a large number of functions have been tabulated by Campbell and Foster[3] and by others, it is easy to select, or construct, examples of functions F the Fourier transforms of which satisfy the above criterion, and which satisfy (3). $F(x) = \sin^m x/x^n$, where m and n are positive integers, $m \geqslant n$, both of them odd or both of them even, is one such function with $v_0 = m$; when $m = 1$, the range of v over which $g(v) \neq 0$ includes $v = 1$. A few other examples are given below.

$F(x)$	$\sqrt{(2\pi)} . g(v)$	v_0
$\dfrac{\sin [a(x^2 + \lambda^2)^{1/2}]}{(x^2 + \lambda^2)^{1/2}}$	$\pi J_0 [\lambda(a^2 - v^2)^{1/2}]$	a
$\cos [a(x^2 + \lambda^2)^{1/2}] - \cos ax$	$-\dfrac{\pi a \lambda J_1 [\lambda(a^2 - v^2)^{1/2}]}{(a^2 - v^2)^{1/2}}$	a
$\dfrac{1}{\Gamma(a + x)\, \Gamma(\beta - x)}$	$\dfrac{(2 \cos 1/2v)^{\alpha + \beta - 2} \{-\frac{1}{2} \exp iv(\alpha - \beta)\}}{\Gamma(\alpha + \beta - 1)}$	π [4]

A more detailed account will be published shortly in the *Journal of the Indian Mathematical Society*.

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[1] Bhatia, A. B., and Krishnan, K. S., *Proc. Roy. Soc.*, A, **192**, 181 (1947).
[2] See Titchmarsh, "Introduction to the Theory of Fourier Integrals", 60 (Oxford, 1937).
[3] Bell Tel. Sys. Tech. Pub. Monograph B584 (1931).
[4] From Ramanujan, "Collected Papers", 216.

Physical Periodicity of the Periodic Table of the Elements in the Light of Statistical Theory

The periodic system of the elements, arrived at originally from the collation of facts of a chemical nature, has since been most strikingly reflected by most of the physical properties of the elements. The melting points, boiling points, coefficients of thermal expansion, atomic volumes, compressibilities, densities, ionization potentials, to mention only a few properties, undergo periodic variations with increasing atomic weight or atomic number.