the penetrating particles will give rise to an average distance between the points of arrival of two mesons on a horizontal plane (at 10 km. below the centre) greater than the distance between the telescopes of our arrangement, and no coincidence will be registered. Thus the exponential variation with atmospheric depth in our experience refers to penetrating showers produced locally.

The rapid increase of the number of slow mesons and neutrons with altitude leads us to associate these particles with local penetrating showers of low energy. The high-energy, long-lived mesons, observed at sea-level and underground, are probably produced also in groups, giving rise to extensive penetrating showers. The cross-section for these processes must be of the same order of magnitude or even greater than the value found by us for local penetrating showers. In all these collisions, energetic secondary nucleons can arise and thus the formation of cascades of nucleons and mesons must be expected.

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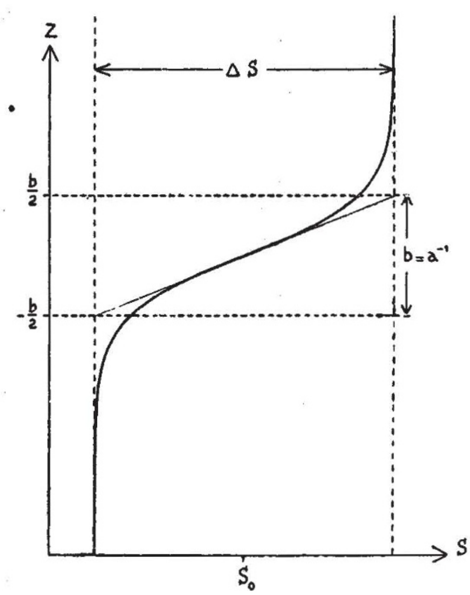
[1] *Phys. Rev.*, **71**, 453 (1947). The radiation producing these showers does not show east–west asymmetry at the altitude of S. Paulo, as was shown by P. Saraiva and G. Wataghin.

[2] "Symposium sobre raios cosmicos", *Acad. Bras. Sci.* (1941).

Internal Waves in Certain Types of Density Distribution

DEALING theoretically with internal gravitational waves in vertically inhomogeneous incompressible fluids, many authors have assumed discontinuities, at certain levels, either in the density or in its first derivative with respect to the vertical co-ordinate, z. Fjeldstad, on the other hand, by using numerical integration, gave an approximate solution of the problem for long waves in certain general, continuous density distributions between a free surface and a rigid bottom.



DENSITY DISTRIBUTION REPRESENTED BY A GRAPH OF THE SPECIFIC VOLUME S (THE ABSCISSA) AGAINST THE VERTICAL CO-ORDINATE z (THE ORDINATE)

It has now proved possible to deal with certain continuous density distributions analytically and find complete solutions which are not restricted to the longer wave-lengths.

The density distribution is of the general type shown in the accompanying graph; the relative variation of density is supposed to be small. The fluid is supposed to be incompressible and to be at rest in the non-perturbed state, whereas, in the perturbed state, an internal wave is propagated in the x-direction.

The perturbation is supposed to be small. If we describe the simple harmonic wave motion by means of a stream function

$$\varphi(x,z,t) = \varphi(z) \exp i(mx - nt),$$

it appears that $\varphi(z)$ may be found with, in general, sufficient accuracy as a solution of the equation

$$\frac{d^2\varphi}{dz^2} + \left(\frac{g\,dS/dz}{c^2 S_0} - m^2\right)\varphi = 0, \qquad (1)$$

where $S(z)$ is the specific volume in the unperturbed state, S_0 is the mean specific volume, $c = n/m =$ velocity of propagation.

As an analytical representation of $S(z)$ we have chosen $S = S_0 + \frac{1}{2}\Delta S \tanh (2z/b)$, where ΔS is the total variation of the specific volume, b is the thickness of the transition layer (see graph). The differential equation (1) then becomes

$$\frac{d^2\varphi}{dz^2} + \left(\frac{g\,\Delta S}{bc^2 S_0 (\cosh (2z/b))^2} - m^2\right)\varphi = 0. \qquad (2)$$

If the fluid extends to infinity both upwards and downwards, we have as boundary conditions, that φ must remain finite or become zero, when $z \to \pm\infty$. This is an eigen-value problem.

The differential equation (2) can be solved by means of certain transformations, the solutions being expressed in terms of hypergeometric series. The eigen-values of m, in terms of c, are given by

$$bm = \sqrt{1 + \frac{bg\,\Delta S}{c^2 S_0}} - (2k+1),\ k = 0, 1, 2, 3, 4$$

. . ., etc., m being positive.

This yields relations between the wave-length $L = 2\pi\lambda$ and the period $T = 2\pi\tau$, which may conveniently be described by

$$\frac{g\,\Delta S}{b\,S_0}\tau^2 = k(k+1)\left(\frac{2\lambda}{b}\right)^2 + (2k+1)\left(\frac{2\lambda}{b}\right) + 1.$$

When $\lambda \to 0$, the period approaches a minimum value for all modes (namely, for all values of k):

$$T_{\min} = 2\pi\sqrt{\frac{bS_0}{g\,\Delta S}} = 2\pi\sqrt{\frac{S_0}{g(dS/dz)_{\max}}}.$$

It appears that any solution $\varphi_k(z)$ has k zeros for finite values of z; for $z \to \pm\infty$ it tends to zero exponentially.

It is easy to extend the theory to the case of a rotating fluid (the atmosphere or the ocean on the rotating earth). In this case, the same relation as exists between τ and λ in the previous (non-rotating) case now exists between τ and $\lambda\sqrt{1 - (2\omega_z\tau)^2}$, ω_z being the angular velocity of rotation around the vertical axis.

Further details will be given shortly in a paper by the author in the *Mededelingen en Verhandelingen van het Koninklijk Nederlands Meteorologisch Instituut*, de Bilt, Netherlands.

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