

For $v(x)$ we obtain the equation

$$\frac{1}{12} x^2 v'' - \frac{1}{2} x v' + v = \left\{ v^{1/2} + \frac{\beta}{12} x^2 \right\}^3.$$

For $v(x) = w(x^2) = w(t)$ we have then

$$\frac{1}{3} t^2 w'' - \frac{5}{6} t w' + w = \left\{ w^{1/2} + \frac{\beta}{12} t \right\}^3. \quad (5)$$

Putting $w(t) = 1 + c_1 t + c_2 t^2 + \dots$ we obtain, by comparing coefficients:

$$c_1 = -\frac{3}{16}\beta, \quad c_2 = \frac{79}{9216}\beta^2, \quad c_3 = \frac{1584}{63700992}\beta^3, \dots$$

Thus we have the solution

$$y = \frac{144}{x} (1 + c_1 x^2 + c_2 x^4 + c_3 x^6 + \dots), \quad (6)$$

and the series converges rapidly, for it is majorized by the geometrical series with the quotient 10^{-2} . We may stop, for example, at the third term.

The shape of our curve is the same as that calculated by Jensen's numerical method. Our solution does not satisfy the condition $y(0) = 1$. We can avoid the difficulty by using an idea of Sommerfeld⁵, namely, we put

$$y \sim \frac{144}{\{1 + (x/a)^2\}^{3/2} a^3} \{1 + c_1 a^2 \{1 + (x/a)^2\} + \dots + c_3 a^6 \{1 + (x/a)^2\}^3\}. \quad (7)$$

This function is asymptotically equal to (6), and the parameter a may be chosen such that the condition $y(0) = 1$ be satisfied.

This asymptotic solution, as well as Sommerfeld's, yields a very good approximation, especially in the exterior parts of the atoms.

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¹ Thomas, L. H., *Proc. Camb. Phil. Soc.*, 23, 542 (1927). Fermi, E., *Atti Acad. Lincei*, (6), 8, 602 (1927).

² Dirac, P. A. M., *Proc. Camb. Phil. Soc.*, 26, 376 (1930).

³ Jensen, H., *Z. Phys.*, 89, 713 (1934); 93, 232 (1935); 101, 141 (1936).

⁴ Gombás, P., *Z. Phys.*, 121, 523 (1943).

⁵ Sommerfeld, A., *Z. Phys.*, 78, 283 (1932).

⁶ Miranda, C., *Atti Soc. Italiana*, 21, 121 (1933).

Brightness of the New Moon

I HAVE been unable to find in the literature any data for the brightness of the new moon. This can be calculated in two independent ways.

(1) The earth receives from the sun at noon about 10^5 lux¹. The albedo value of the earth is about 45 per cent². The brightness of the earth, as seen from a great distance, is therefore 4.5×10^4 apostilb (equivalent metre candles) or 1.43×10^4 c./m.². The cross-section of the earth is 1.27×10^8 km.², or 1.27×10^{14} m.². The candle-power of 'full earth' is therefore 1.82×10^{18} c. The distance earth to moon being 3.84×10^8 m., the latter receives from the earth 12.3 lux. The illumination on the earth at full moon being only 0.2 lux³, the illumination-level on the moon at 'full earth' is consequently about sixty times better.

The albedo value of the moon is about 7.3 per cent²; therefore, the brightness of the new moon is 0.86 apostilb or 2.7×10^{-5} stilb (candle per sq. cm.).

(2) The earth receives at full moon 0.2 lux. The surface of the earth is 15.1 times that of the moon.

The earth reflects 6.2 times as well as the moon. Because the sun sheds the same number of lux on the earth as on the surface of the moon, the moon receives at full earth $15.1 \times 6.2 \times 0.2$ lux or 18.7 lux. The moon reflects 7.3 per cent. The brightness of the new moon is thus 1.37 apostilb or 3.6×10^{-5} stilb.

Both methods lead thus to a value of about 3×10^{-5} stilb or about 1 apostilb. The brightness of full moon is about 0.3 stilb or 10,000 times higher.

With the aid of a formula given by P. J. Bouma³ the brightness of the new moon can be converted into magnitudes and is equal to -2.4 , whereas for the full moon the magnitude is -12.5 .

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¹ Geiger, H., and Scheel, K., "Handbuch der Physik", 19, 446.

² Schönberg, E., *Erg. exakt. Naturwiss.*, 5, 6 (1926).

³ Philips Tech. Rev., 5, 308 (1940).

Darkening of Red in Protanopes

THE degree of darkening of the Ilford Spectrum Red colour-filter was measured for twelve protanopes and compared with its brightness for eighteen deuteranopes and several hundred normal subjects. The light transmitted by the red filter from a daylight source was mixed with light transmitted by the Ilford Spectrum Green filter and matched for brightness against light transmitted by the Ilford Spectrum Yellow filter by varying the intensity of the yellow. The mean brightness-level for the eighteen deuteranopes was the same as for normal subjects for all red-green mixtures. For the twelve protanopes the red light was only 15.5 per cent as bright on the average as for normal subjects; but the brightness-level steadily increased as green was added, until it reached the normal level at the pure green end.

At the extreme red end of the scale, the scatter of brightness measurements for the twelve protanopes did not overlap the scatter for the eighteen deuteranopes. No calculations are necessary to show that we are dealing with two groups of subjects which are statistically distinct.

This experiment confirmed the view put forward previously¹ that the Ishihara Test for colour blindness fails to distinguish clearly between protanopes and deuteranopes. Of the eighteen deuteranopes who did the present experiment, fourteen also did the Ishihara Test, which failed to distinguish five of them clearly from protanopes. Similarly, nine of the twelve protanopes also did the Ishihara Test, and it failed to distinguish eight of them unequivocally from deuteranopes, although the evidence for this distinction based on actual measurements of the darkening of the red is very clear indeed. It is interesting that Geddes², assuming the validity of the Ishihara Test, makes a separate class ('incompletely red-green blind') for those subjects not decisively placed as protanopes or deuteranopes (that is, scoterythrous or photerythrous) by that test.

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¹ Pickford, R. W., *Nature*, 153, 656 (1944).

² Geddes W. R., *Brit. J. Psychol.*, 37, 31 (1946).