of ln p_1^0/p_2^0 appropriate to the boiling point of the mixtures are plotted against N_1. The equation applicable to isobaric data when the components differ considerably in boiling point is rather more complex.

The derivation of equation (1) and examples of its use in the investigation of the systems thiophene – benzene and ethylene dichloride – benzene will be published elsewhere.

I am indebted to Dr. R. P. Linstead, director of the Chemical Research Laboratory, for permission to publish this note.

E. F. G. HERINGTON

Chemical Research Laboratory,
Teddington, Middlesex.
July 11.

[1] Carlson and Colburn, *Ind. Eng. Chem.*, 34, 1533 (1942).

Hydrodynamics of Non-Newtonian Fluids

IF the assumption is made that a fluid is incompressible, then the most general possible relations between the stress components $(t_{xx}, t_{yy}, \ldots)$ and the strain-velocity components $(a, b, \ldots)$ can be obtained as

$$t_{xx} = 2\Theta.a + 2\Psi.A' + p \ldots,$$
$$\text{and} \quad t_{yz} = \Theta.f + \Psi.F' \ldots \qquad (1)$$

$(a, \ldots f, \ldots)$ are given, as in classical hydrodynamics, by

$$a = \partial u/\partial x \ldots \text{ and } f = \partial v/\partial z + \partial w/\partial y \ldots,$$

where (u,v,w) are the components of velocity at a point (x, y, z). $(A_1', \ldots F_1', \ldots)$ are given, in terms of $(a, \ldots f, \ldots)$, by the relations

$$A' = a^2 + \tfrac{1}{4}(g^2 + h^2) \ldots$$
$$\text{and} \quad F' = \tfrac{1}{2}gh + (b + c)f \ldots \qquad (2)$$

p is a hydrostatic pressure. Θ and Ψ are arbitrary functions of the strain-velocity invariants K_2 and K_3, where

$$K_2 = ab + bc + ca - \tfrac{1}{4}f^2 - \tfrac{1}{4}g^2 - \tfrac{1}{4}h^2$$
$$\text{and } K_3 = abc + \tfrac{1}{4}fgh - \tfrac{1}{4}af^2 - \tfrac{1}{4}bg^2 - \tfrac{1}{4}ch^2.$$

The invariant $K_1 (= a + b + c)$ is zero, since the fluid is incompressible.

The stress *vs.* strain-velocity relations (1) are somewhat similar to those obtained by Reiner[1] for a compressible fluid.

For a Newtonian fluid, $\Psi = 0$ and Θ is a constant—the viscosity of the fluid. For a non-Newtonian fluid, Θ depends on the strain-velocity invariants K_2 and K_3. It appears, from (1), that for a general, incompressible fluid, there is a second physical parameter Ψ which must be specified in order to define completely the flow properties of the fluid. This, again, may be a constant, or may depend on the strain-velocity invariants K_2 and K_3.

Equations (1) have been applied to calculate the forces which must be exerted on the boundary surfaces of a general, incompressible fluid in the following two cases.

(i) A cylindrical mass of fluid is subjected to a single torsional motion, in which each point of the fluid moves in a circular path about the axis of the cylinder, in a plane at right-angles to it, its angular velocity ψz being proportional to its distance z from one end of the cylinder.

(ii) A mass of fluid is contained between two coaxial cylinders, which are rotated with respect to each other about their common axis, and each point of the fluid moves in a circular path about this axis, in a plane at right-angles to it, with an angular velocity Ω, which depends only on the distance from the axis.

In case (i), it is found that in order to produce the specified motion of the fluid, both azimuthal and normal surface tractions must be exerted on the plane ends of the cylinder. If Ψ is constant, then the normal surface traction Z which must be exerted at a point at a distance r from the cylinder axis is given by

$$Z = \tfrac{1}{4}\Psi\psi^2 (3r^2 - a^2), \qquad (3)$$

where a is the radius of the cylindrical mass of fluid.

In case (ii), it is found that in order to produce the specified motion of the fluid, azimuthal surface tractions must be exerted on the cylindrical surfaces of the fluid, and normal surface tractions Z must be exerted on the plane-free surfaces. If Ψ is constant, the value of Z at a distance r from the common axis of the cylinders is given by

$$Z = -2\Psi(\Omega_1 - \Omega_2)^2 \left(\frac{a_1^2 a_2^2}{a_1^2 - a_2^2}\right)^2 \frac{1}{r^4}; \qquad (4)$$

a_1 and a_2 are the radii of the inner and outer confining cylinders and Ω_1 and Ω_2 are their angular velocities.

In the calculations of both cases (i) and (ii) it is assumed that centrifugal and gravitational effects can be neglected. It is seen, from (3) and (4), that if $\Psi = 0$, $Z = 0$ for both systems. The effects which arise from the non-vanishing of Ψ are similar to those obtained by Weissenberg[2] with a variety of fluids. It does not appear, however, that his explanation of these effects can be a valid one, since it depends on the presence of an elastic component in the stress *vs.* strain relations for the fluid, which could not make itself evident in any experiment on steady-state flow. From the point of view of phenomenological theory, such effects can arise in a material which is not visco-elastic. Whether or not materials which show this effect are, in practice, visco-elastic is a matter which must be decided either on experimental grounds or by studying the dependence of Ψ and of rigidity on the constitution of the fluid.

This work was carried out as part of a programme of fundamental research undertaken by the board of the British Rubber Producers' Research Association, while I was a guest worker at the National Bureau of Standards, Washington, D.C. It is hoped to publish a fuller account elsewhere.

R. S. RIVLIN

British Rubber Producers' Research Association,
Welwyn Garden City,
and
National Bureau of Standards,
Washington, D.C.

[1] Reiner, M., *Amer. J. Math.*, 67, 350 (1945).
[2] Weissenberg, K., *Nature*, 159, 310 (1947).

A Long-Term Plan for the Nile Basin

IN an article on "The Nile Basin", vol. 7, in *Nature* of August 16, p. 215, Dr. C. E. P. Brooks directs attention to the fact that the theory underlying what we have called 'century storage' is not given in the book. This was omitted because it was thought better not to make a book dealing with Nile projects more difficult by including in it complicated theoretical investigations. It is the intention to publish a full account of these separately; but it may be opportune to give here a very short summary of what has been done, and also to clear up a point raised by Dr. Brooks as to the correctness of the