This investigation was suggested by Prof. W. G. Burgers.

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Stress-Optical Properties of Rubber

CONSIDERABLE progress in the quantitative description of the mechanical properties of rubber has taken place in recent years by the consideration of a rubber as a network of statistically kinked long-chain molecules. A treatment of the optical properties of strained rubber on this basis, assuming the links of the molecular chain to be themselves optically anisotropic, has been developed by Kuhn and Gr $\ddot{u}$ n¹ for the special case of a simple elongation, and by me² for the general homogeneous strain. In the latter case, the rubber has three principal refractive indices, n_1 , n_2 and n_3 , corresponding to the three principal axes of the strain ellipsoid.

For light propagated along a principal axis, the birefringence is found to be

$$n_1 - n_2 = C_1 N (\lambda_1^2 - \lambda_2^2), \quad (1)$$

where N is the number of chains per c.c. of the network, and λ_1 , λ_2 , λ_3 are the principal extension ratios. The stresses t_1 , t_2 and t_3 are related to the principal extensions by equations of the type

$$t_1 - t_2 = NkT (\lambda_1^2 - \lambda_2^2). \quad (2)$$

Hence the birefringence in a principal plane is proportional to the difference of the principal stresses, and to the difference of the squares of the principal extension ratios.

By stretching a rubber sheet in two perpendicular directions in such a way as to produce a homogeneous

Fig. 1. DEPENDENCE OF DIFFERENCE OF PRINCIPAL STRESSES AND BIREFRINGENCE ON DIFFERENCE OF SQUARES OF PRINCIPAL EXTENSION RATIOS

Fig. 2. CROSSES, EXPERIMENTAL RESULTS; CIRCLES, CALCULATED FROM MOONEY'S THEORY, WITH $G = 2.0$, $K = 0.2$

strain, it has been possible to test these theoretical predictions. The closest agreement was obtained when the rubber was swollen with a solvent. Fig. 1, which refers to a pure gum natural rubber swollen in paraffin to about twice its original volume, confirms equations (1) and (2) fairly precisely.

In these experiments the stress (t_3) normal to the sheet was zero. In this case, the stress t_1 in the direction corresponding to λ_1 is, according to the theory,

$$t_1 = NkT (\lambda_1^2 - \lambda_3^2), \quad (2a)$$

where λ_3 is the extension ratio in the direction normal to the surface of the sheet. However, on plotting t_1 against $(\lambda_1^2 - \lambda_3^2)$, the points fall, not on a straight line, but on a series of arrays (Fig. 2). In any one of these arrays one force f_1 (corresponding to the stress t_1) is constant, while the other force f_1 varies.

A physical explanation of this departure from the behaviour predicted by the molecular theory has not been found. It is, however, exactly the type of behaviour to be expected from the more general theory of elasticity developed by Mooney³, on the assumption that Hooke's Law is obeyed in shear. From Mooney's theory, the principal stresses are found to be related to the extension ratios by equations of the type

$$t_1 - t_3 = (G + K\lambda_2^2) (\lambda_1^2 - \lambda_3^2). \quad (3)$$

With a suitable choice of the constants G and K , the graphical solution of these equations leads to results which, when plotted as in Fig. 2, agree remarkably well with the experimental data.

For the dry rubber, on the other hand, Hooke's Law was not obeyed in shear, and neither the molecular theory nor Mooney's theory accounted quantitatively for the experimental data, though Mooney's theory provided the closer approximation.

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