## THE AIRY INTEGRAL

**British Association for the Advancement of Science**
Mathematical Tables. Part-Volume B : The Airy
Integral, giving Tables of Solutions of the Differential
Equation y″ = xy. Prepared by J. C. P. Miller.
Pp. B.56. (Cambridge : At the University Press,
1946.) 10s. net.

THE tabulation in this volume is mainly con-
cerned with the function $Ai(x)$ and its derivative,
and before discussing the actual details of the tables,
it is instructive to recall the history of the function.

A little over one hundred years ago, in calculations
relating to the light-intensity in the neighbourhood
of a caustic, Airy was confronted with the integral

$$W = \int_{0}^{\infty} \cos \tfrac{1}{2} \pi \ (w^3 - mw) \ dw,$$

and succeeded in producing a 5-decimal table for
values of $m$ ranging between $\pm 4$, later extended
to $\pm 5 \cdot 6$.

It can be deduced that $W$ satisfies the differential
equation

$$\frac{d^2 W}{dm^2} = - \frac{1}{12} \pi^2 \, m \, W,$$

and hence that the problem of computing it is
effectively that of computing the solutions of the
second order differential equation

$$\frac{d^2 y}{dx^2} = xy.$$

The form of this last equation suggests that its
solutions are closely allied to Bessel functions of a
certain order, and denoting them by $Ai(x)$ and $Bi(x)$,
it may be shown, for example, that when $x$ is real
and positive,

$$Ai\,(-x) = \tfrac{1}{3} \, x^{\frac{1}{2}} \{ J_{-\frac{1}{3}} \, (\xi) + J_{\frac{1}{3}} \, (\xi) \}$$
$$Bi\,(-x) = (\tfrac{1}{3} \, x)^{\frac{1}{2}} \{ J_{-\frac{1}{3}} \, (\xi) - J_{\frac{1}{3}} \, (\xi) \}$$

where $\xi = \tfrac{2}{3} x^{3/2}$, with similar results for positive
arguments in terms of modified functions.

Recently the demand for tables of the Airy integral
has revived ; a revival closely connected with the
simplicity of the differential equation above, and in
virtue of which the British Association, through its
tables committee, has prepared this volume (issued
as Part-Volume $B$, in conjunction with a volume
dealing with the tabulation of Legendre polynomials).

It is provided with a substantial introduction
discussing the history of the function, the formulæ
and equations involved, and the methods of com-
puting and checking. This occupies some fifteen
pages and is followed by tables of $Ai(x)$ and $Ai'(x)$,
for the range $x = - 20$ to $x = 2$, at intervals of
$0 \cdot 01$, the entries being to eight decimal places, with
second differences.

Subsidiary tables of the logarithms, zeros and
turning values of the functions are also included,
and similar results for the $Bi$ functions for the range
$- 10 \cdot 0 \ (0 \cdot 1) + 2 \cdot 5$ are provided. Finally there are
eight pages of tables of auxiliary functions, with
definitions and formulæ.

The whole book is beautifully produced and does
its authors great credit. It is interesting, in con-
clusion, to note that Airy's original computations, in
spite of the exceedingly laborious work which they
involved, were almost entirely devoid of error.

J. H. PEARCE

## MAP PROJECTIONS

**Map Projections**
By George P. Kellaway. Pp. viii + 128. (London :
Methuen and Co., Ltd., 1946.) 10s. 6d. net.

WE welcome a book whose "aim is to provide
students of geography with a logical treatment
of the commoner map projections". A careful study
leaves no doubt of its value as a contribution to the
teaching of the subject : the plan is well conceived
and skilfully executed.

The earth sphere, latitude and longitude are com-
pressed into four pages, which lead to discussion of
the fundamental problem of representation of the
earth on flat maps. There is a double classification
based on : (1) nature—perspective, geometrical and
conventional ; (2) construction—on plane, cylinder or
cone.

Part 1 begins with polar maps : the order gnomonic,
stereographic, orthographic is surprising ; there is
something to be said for beginning with the sphere
as ordinarily seen. Cylindricals follow, and here the
discussion of Cassini's is particularly welcome though
the construction is not detailed, probably because it is
too difficult for those for whom the book is intended.
Conicals, ending with Bonne's, complete the geo-
metrical types. Conventionals open with Sanson-
Flamsteed. Aitoff receives due notice as an improve-
ment on Mollweide.

The plan of treatment in each case is : (1) mathe-
matical basis ; (2) statistical examination of proper-
ties at regular latitude intervals ; (3) summary of
general properties ; (4) limitations ; (5) construction.
The statistical treatment leads to the study of limita-
tions. This is very thoroughly done. Perhaps, after
full treatment in one case and a beginning in others,
something might, with advantage, have been left
for the student to do ; on the other hand, much
labour has provided some very acceptable reference
material.

Part 2 deals with equatorial zenithals and obliques.

The outstanding impression is that the author is
deeply concerned, and quite rightly, that the student
should realize the importance, in dealing with approxi-
mations, of considering the degree of accuracy
attained. One would like to think of non-
mathematical geographers at school or university
being at home with the mathematical work, in these
days of numerical trigonometry to School Certificate.
To follow the manipulative work would be a severe
tax ; at worst, the student can take it as read, pay
due attention to the statistical data and construct
according to instructions. No student can really
be said to have studied map projections without
constructive work.

The book ends with an admirable discussion of the
use of various projections and a collection of repre-
sentative questions set by university examining
boards. Could we not give them a lead in setting
questions concerned with travel, and would it not be
worth while for examiners to ask a candidate to con-
struct rather than write about projections, at least
sometimes ?

In conclusion, the reviewer has read the book with
real pleasure : he feels no hesitation in recommending
its careful study by those seeking a stimulating text-
book on the subject or a worthy addition to the
geographical library. He congratulates author,
publisher and printers on an excellent production.

A. HINCKLEY