year period 1936–40, Pretoria serving as station A. By drawing a map showing lines of equal correlation, it was possible to determine the value of $r_{(Aq)}$ at all points on a circle of roughly one thousand miles in diameter covering the greater part of the Union of South Africa, and to express $r_{(Aq)}$ analytically in terms of its harmonic components. Similarly, by drawing a series of maps showing lines of equal correlation of *simultaneous* pressures, it was possible to express $r_{(q,s)}$ in terms of a double Fourier series involving the two variables q and s. By substituting the values thus found in (5), one is able to obtain the value of $K(s)$ in terms of its Fourier components, and hence the value of R_z by means of equation (6).

The value actually found for the month of July was

$$R_z = 0\cdot916 \pm 0\cdot003,$$

and this value represents an accuracy of prediction which compares most favourably with the accuracy obtained by competent meteorologists in drawing prebaratic or prognostic charts.

However, the above value of R_z by no means represents the ultimate possibilities of this method. The maximum value of R_z, which in a previous publication[1] we denoted by M_z, is attained only when all possible controls are included, which implies that pressures at *all* points in the atmosphere measured at *all* times from $t = -\infty$ to 0, should be included in the regression equation. This in turn means that the single integration in (5) and (6) should be replaced by a fourfold integration with respect to space and time to obtain the value of M_z and hence the maximum reliability of prediction.

From the example quoted it will be evident that this method, in which a system of linear equations is transformed into a single integral equation, opens up a very wide field of research, and by the systematic investigation of the value of M_z when z ranges, say, from 6 hours to 6 months, a final verdict may be reached concerning the possibilities and limitations of both medium and long-range weather forecasting.

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[1] Schumann, T. E. W., *Quart. J. Roy. Met. Soc.* (July 1944).

"Turbulent Flow in Alluvium"

PROF. C. M. WHITE, in commenting on Mr. Gerald Lacey's letter on "Turbulent Flow in Alluvium" published in *Nature* of August 3, p. 166, stated, "Mr. Gerald Lacey has discussed the dimensions of rivers flowing in beds of incoherent alluvium . . ."

This is not quite correct. Mr. Lacey's original formulæ of 1930[1]—which remain substantially the same to-day, sixteen years later—were based on a considerable mass of accurately measured data observed in canal channels in which the discharges were maintained nearly constant. Since then, much more data have been collected—in many cases at intervals throughout the year—by specially trained staff, in channels which do not change appreciably from year to year, and are run with almost constant discharges. These data have been statistically analysed and it has been found that the more data that become available, the better the agreement with the Lacey formulæ.

The formulæ presented by Mr. Lacey in his letter are not new, except in their form of presentation, and were inherent in his original formulæ.

What he has done is to substitute

$$f_{SV} = 48\sqrt{SV} \propto (\nu g)^{1/6}$$

as a sand factor, in place of the earlier

$$f_{VR} = 1\cdot155\ V^2/R \propto g.$$

Lacey makes these equal numerically at regime, but they are different dimensionally, due to gravity and kinematic viscosity being omitted for simplicity; because they were designed for use by practical engineers.

All Lacey's formulæ are based on two fundamental relationships

$$\frac{gDS}{V^2} \text{ or } \left(\frac{V^*}{V}\right)^2 \text{ and } \frac{V^2}{\frac{1}{2}gw} \text{ (the Froude number for width),}$$

$$\text{and } \frac{gDS}{V^2} \text{ and } \left(\frac{VD}{\nu}\right) \text{ (Reynolds' number).}$$

Surely Prof. White does not suggest that "On algebraically combining two such formulæ one could prove anything"!

Next as regards what should be treated as "independent variables": Prof. White has adopted Q, g, Vs—the terminal speed of a typical particle falling through water—and N—the quantity of solids expressed as a fraction of the water flow; and he has selected the area of cross-section at bank-full stage as a dependent variable. He then eliminates N by grouping rivers in which the charges, *as measured*, vary between 1/1,000 and 1/5,000; but he has not stated how such measurements were observed, nor has he yet presented the data of the ten selected rivers on which his formulæ were based.

Experiments carried out by me at Poona[2] showed that the rate of deposition of sand of various grades in turbulent water varies as $(N.Vs)$—that is to say, a heavy charge of silt gives the same rate of deposition as a correspondingly lighter charge of medium sand—so that if the charge—N—varied between 1/1,000 and 1/5,000, either bed movement must have been ignored—as seems probable, because no method of measuring movement of bed sand outside a research station has yet been devised—or else a wide range of charge must have seriously vitiated the results—unless, of course, there was so little movement that charge was an unimportant factor. This is what Lacey assumed for his regime conditions.

There is little difference, therefore, between Lacey's original selection of variables and White's.

		Lacey's original independent variables	White's independent variables
1	Q	Accurately measured practically constant discharges.	Discharge observed at bank-full stage assumed to represent normality.
2	N	Regime charge—the minimum charge associated with a fully active bed.	A range of charge from 1/1,000 to 1/5,000 assumed not to affect results appreciably.
3	f, Vs	f, a sand factor originally linked with V^2/R but later with $\sqrt{SV}$.	Vs, the terminal velocity of what is called a typical particle.

As regards Vs, experience in India shows that the material exposed on the bed of a channel is continually varying, both as regards grade (Vs) and charge(N). That, in fact, changes in N and Vs represent the 'mechanism of adjustment' to meet changing flow conditions. Thus N and Vs are highly dependent variables, the former of which cannot be measured with any degree of accuracy outside a research station and the latter only with difficulty—because samples of bed material have to be taken at the same time as the area of section, discharge and water temperature are observed.

It may be argued that (SV) is an equally poor criterion of independence—on the grounds that S and V are both dependent variables—but S can only alter very slowly, and experience shows that with constant discharge, but varying charge and grade, $V = (Q/a)$ also alters slowly, and that (SV) alters still more slowly. Thus, though S and V depend on rainfall, the material washed into the river, the temperature of the water, and the variations in all of these—which cause 'trading' of material during alternating conditions of scour and accretion—yet (SV) is the best measure of the integrated effects of sand charge and grade on a long-term basis and probably also on a short-term basis; because it is easily measurable and is proportional—after eliminating the effects of discharge—to the overall effects of charge, grade, shape and specific gravity of particles, and water temperature.

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Aug. 29.

[1] Lacey, G., *Proc. Inst. Civ. Eng.*, 229 (1930).

[2] Inglis, C. C., Ann. Rept. Tech. Central Irrigation and Hydrodynamic Research Station, Poona, India (1941–42).

PROF. C. M. WHITE, in his valuable and constructive comments on my new flow equations[1], has raised certain questions, which, if the subject of alluvial transport is to be further advanced, demand a reply.

The Lindley theorem[2] of 1919, of which my 1930 equations were a natural outcome, asserted that for a given discharge, particle size, and transported load, the dimensions of a channel flowing uniformly in an unlimited medium of its own self-transported alluvium are, ultimately, uniquely determined. The dependent variables are therefore P, R and S, the wetted perimeter, hydraulic mean depth, and water surface slope.

The conditions postulated are ideal and more easily achieved in the laboratory than in the field. On well-established perennial canals, the conditions in respect of discharge, particle size, and transported load are tolerably fulfilled. The engineer, however, by somewhat arbitrarily assigning width, depth and slope, when constructing his canals and making his excavations in the natural soil, presents Nature with the immediate task of modifying the designed depth of water, followed by a further adjustment in depth and slope which is also accompanied by modifications in the width if the soil is friable and permits of this taking place.

The 1939 equations of Prof. White are effectively an application of the Lindley theorem to rivers, and in particular to those rivers in the alluvial plains which by a cycle of erosion and accretion have generated their own cross-sections and established a slope which can correctly be regarded as a dependent, as opposed to the sensibly constant and independent variable of the slope of shingle and boulder torrents, of which the actual size of bed particle exposed at any given time is a dependent variable and a function of the discharge intensity. Rivers in the plains generate their own boundaries and slopes, but, owing to the admixture of fine adhesive particles and 'ageing', the banks and portions of the bed are frequently far from incoherent. As a result, if the gross slope is measured over many miles, this slope is not a simple dependent on the cycle of discharges and the particle size, but is complicated by the addition of other factors leading to loss of energy and an increase in the slope. The poor correlation of Prof. White's slope equation is probably due mainly to this cause.

The dependent variables of P, R and S having been assigned, all other variables, known, or unknown, are independent, and Prof. White's method of dealing with them is highly ingenious and effective. Failing measurement of the transported load, N, we are forced either to treat it as constant, or to adopt a criterion in which both particle size and load are implicit.

Prof. White has directed attention to the impropriety of combining two empirical equations algebraically. With his contention I fully agree, but would submit that when the two equations have each a high correlation, and the merit of extreme simplicity in the powers, the ends may justify the means, and serve to demonstrate the truth of past experience that more than one advance has owed its existence to a leap in the dark, ending happily on firm ground.

The risk, to which Prof. White has referred, that one may derive two empirical equations "which look different but which do in fact state the same thing though containing different errors of field measurement", is one that all unwittingly may run.

The dimensionless number of Prof. White

$$ag^{2/5}/Q^{4/5}$$