Role of Inertia in Hydrodynamic Lubrication

ON the basis of the hydrodynamic theory of lubrication presented by Prof. Osborne Reynolds in 1886[1], plane parallel surfaces in relative motion are incapable of developing a load-supporting oil film. This theory requires that the bearing surfaces be inclined to one another in the direction of motion. Recently, Mr. A. Fogg of the National Physical Laboratory reported[2] load capacities for parallel thrust surfaces, operating at high speeds, that were comparable to those obtained with conventional Michell or Kingsbury type thrust bearings. Similar results have also been noted by other researchers[3].

Mr. Fogg explains these anomalous observations on the basis of a temperature gradient along the oil film in the direction of relative motion. He likens the expansion of the lubricant accompanying this temperature gradient to an equivalent hydrodynamic oil wedge. This theory is shown to be in qualitative agreement with the observed data by a process of reasoning by analogy.

A mathematical analysis of the development of a load-supporting oil film by lubricant expansion has been made. As Mr. Fogg predicted, the expansion of the lubricant as it passes through the bearing is significant, particularly in the case of parallel surfaces. However, the results of this analysis are not in quantitative agreement with Mr. Fogg's observation that at high rotative speeds a pair of parallel plates will carry very nearly the same load as a comparable tilting pad bearing. The analysis rather shows the load capacity of parallel surfaces to be of the order of 1/5 to 1/10 that for an equivalent tilting pad bearing, the exact value depending upon the temperature rise and coefficient of expansion obtaining. Although Mr. Fogg's theory nicely explains why parallel thrust surfaces, for use at ordinary speeds, are generally designed to support a unit load of approximately 50 lb. per sq. in. while tilting pad bearings are designed to carry a load of the order of 500 lb. per sq. in., it does not explain the excellent performance of parallel surfaces at high rotative speeds. As Mr. Fogg points out in his paper[2], the influence of the coefficient of expansion of the lubricant should be included in the analysis of slider and journal bearings which operate with a large rise in temperature.

The unexpected load capacity of parallel plane surfaces can be qualitatively explained on the basis of the inertia-induced pressure developed in an oil film at high rotative speeds. Here the density of the lubricant is the physical property of interest. The inertia of the lubricant has been considered negligible in nearly all hydrodynamic lubrication investigations since the time of Osborne Reynolds. The relatively good agreement between theory and experiment for ordinary slider and journal bearings operating at slow or moderate speeds has amply justified the exclusion of inertia terms. The late Dr. A. Kingsbury[4] has stressed the need for an investigation of the influence of inertia upon the hydrodynamic characteristics of a bearing. However, a consideration of the magnitude of the centripetal component of acceleration led him to conclude[5] that the inertia of the oil in the film was of negligible importance even for bearings operating at high rotative speeds.

In addition to the centripetal component of acceleration, several other components act upon a particle of oil confined between two parallel rotating disks. These include linear, angular and Coriolis acceleration components. An examination of the hydrodynamic equations including inertia terms qualitatively accounts for the observed load capacity of parallel rotating disks. The fact that such a bearing carries an appreciable load only at relatively high speeds is in agreement with the inertia theory of lubrication.

A bearing specially designed to take full advantage of the inertia effect *at all speeds* has been built and tested. This device gives appreciably different curves when $\left(\frac{ZN}{P}\right)$ is plotted against the coefficient of friction at several low speeds, where Z is the absolute viscosity of the lubricant in centipoise, N is the speed in r.p.m., and P is the unit load on the projected area of the bearing. Such a multiplicity of friction curves is contrary to ordinary hydrodynamic theory but is in agreement with the inertia theory of lubrication. It is to be expected that careful friction tests of plane parallel disks, thrust bearings of fixed inclination and Michell- or Kingsbury-type bearings will indicate a similar, although smaller, dependence upon speed at high rotative speeds.

A full report of this investigation will soon be published elsewhere.

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[1] Reynolds, O., *Phil. Trans. Roy. Soc.*, **177**, 157 (1886).
[2] Fogg, A., *Engineering*, **159**, 138 (1945).
[3] Newbigin, H. T., *Proc. Inst. Civil Eng.*, **196**, 223 (1914).
[4] Kingsbury, A., *Trans. Amer. Soc. Mech. Eng.*, **50**, 6 (1928).
[5] Kingsbury, A., *Trans. Amer. Soc. Mech. Eng.*, **53**, 59 (1931).

THE letter from Messrs. Shaw and Strang states that the behaviour of lubricated parallel surfaces, which I reported in a paper to the Institution of Mechanical Engineers in January 1945, is mainly due to the inertia and not, as I suggested, to thermal expansion of the lubricating fluid. While agreeing that the influence of thermal expansion must be included in the analysis of film lubricated bearings, they suggest that its effect is only of the order of 1/5 to 1/10 of a corresponding geometrical wedge. I should like to ask on what basis this comparison has been made. Has it, as seems probable, been made on the basis of a given mean film thickness with corresponding relative velocity and oil viscosity? If so, I suggest that this is not the correct basis for comparison, since the parallel surface bearing will run with a much lower film thickness before contact takes place than will a fixed taper or tilting pad bearing, for a given degree of surface finish. The fixed-taper surface will obviously fail by contact at the trailing edge when the mean film thickness is relatively large, while experiment shows that the tilting pad bearing, also, fails in the same way. It has, in fact, been shown experimentally that, if taper bearings are cautiously failed a number of times so that the bearing metal is wiped away from the trailing edge without deterioration of the surface condition and a parallel portion established along the surface, the load capacity is thereby increased.

Since my original paper on this subject, further work has shown that the parallel surface thrust bearing functions similarly at low speeds, and, in a number of cases examined, the coefficient of friction $-ZN/P$ curve is substantially the same for the speed range 1,000 to 20,000 r.p.m. In a few cases, with bearing pads of longer arc length, there is some indication that high speed gives a small reduction in friction at constant values of ZN/P well removed from the limiting values. Thus, using the argument of Shaw and Strang that the inertia theory should give a multiplicity of friction $-ZN/P$ curves, and with which I agree, these later experiments indicate that inertia has, at most, a second order effect.

During my earlier work, which covered the speed-range 10,000–20,000 r.p.m., the possibility of inertia effects accounting for the behaviour was considered and rejected because there was no multiplicity of coefficient of friction $-ZN/P$ curves. It seemed, at first, the obvious explanation, and it was only after much searching that the thermal expansion theory was put forward. Qualitatively, it seemed the only explanation which agreed with the experimental results, and the letter from Shaw and Strang does not convince me otherwise. I shall, however, look forward with great interest to seeing their promised full report on their investigations.

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Spectroscopic Arguments for Isomeric Structures in α-Chloro-Acids

IN a recent paper, Renard[1] has applied a method by which the qualitative analysis of the hydrolysates of proteins seems easily realized. The first step of the process consists in transforming the α-mono-amino mono-carboxylic acids resulting from hydrolysis into α-chloro-acids; the latter having been separated into different groups by fractional distillation, Raman analysis is used to identify the compounds. The fundamental assumption made by Renard is that the spectrum of the mixture results from the superposition of the spectra of the constituent acids.

It is perhaps worth while pointing out that the Raman spectra published by Renard have certain characteristic features which indicate that this assumption is unjustifiable. Thus we observe that in each spectrum of the mono-chloro acids (propionic, valeric, isovaleric, caproic, isocaproic, β-methyl valeric) two large, diffuse bands at about 1,660 and 1,730 cm.$^{-1}$ occur, one of which is presumably due to association. By analogy with the behaviour of other acids[2] we suggest that the latter should be correlated with the C=O frequency in the monomeric molecule and the former with the same frequency in the dimer. The intensities are difficult to evaluate on the enlargements, but 1,660 cm.$^{-1}$ is clearly the most intense line, which indicates considerable polymerization.

It should also be noted that a band situated at about 1,450 cm.$^{-1}$ is a double one. In some cases, we have been able to evaluate the distance between the components at about 20 cm.$^{-1}$. This could possibly be due to the splitting of the simple C−O frequency in the dimer. Many other lines seem to be double, but their exact interpretation would be difficult with the data available at present.

Furthermore, when acids A and B are mixed, the existence of the following equilibria should be considered, namely:

$$2A \rightleftharpoons A_2$$
$$2B \rightleftharpoons B_2$$
$$A + B \rightleftharpoons AB.$$

A and B being very similar, it seems that AB must play an important part and give rise to new Raman lines the frequencies of which lie between those of A_2 and B_2. Thus, Renard's assumption on the additivity of the spectra seems premature, and for this reason alone no qualitative analysis can be undertaken before this point has been cleared up. May we remark that this is only one of the numerous points which, in our opinion, are open to criticism in Renard's paper. Among them, we might mention the precision of the measurements, which is illusory, and the contradiction between the intensity ratios of the spectral lines, which do not remain constant in the different mixtures.

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[1] Renard, M., *Mem. Soc. Roy. Sci. de Liège*, **7** (1946).
[2] Davies, M. M., and Sutherland, G. B. B. M., *J. Chem. Phys.*, **6**, 755 (1938). Herman, R. C., and Hofstadter, R., *J. Chem. Phys.*, **7**, 460 (1939).

Fisher's "Problem of the Nile"

THE following problem is of central importance in the theory of statistical estimation: k unknown parameters are to be estimated from a sample S consisting of n independent observations from the same parent population; how far can the information in S, relevant to the estimation of the k parameters, be confined to k degrees of freedom? Fisher[1] has shown that a complete solution is possible