ASSOCIATED LEGENDRE FUNCTIONS

Tables of Legendre Associated Functions

By Zaki Mursi. (Fouad I University: Faculty of Science, No. 4.) Pp. viii+286. (Cairo: E. and R. Schindler, 1941.)

It is indeed curious that, after a lapse of nearly forty years, during which the tables of Tallqwist, first published at Helsingfors in 1906, have remained the only example of tabulation of the associated Legendre functions, two new sets of tables should appear almost simultaneously from entirely different sources. The recent appearance of this volume, actually published in Cairo in 1941, coincided almost exactly with that of another, bearing the same title, prepared by the Mathematical Tables Project in New York in 1945. Fortune has, however, decreed that both sets of tables shall prove to be valuable additions to the world of mathematical literature, for not only are they both more comprehensive than Tallqwist's collection, but in different ways, and much information is to be found in each which is not in the other.

The associated Legendre functions, denoted respectively by $P_n^m(x)$ and $Q_n^m(x)$ where m is the order and n the degree, may be defined as suitably normalized solutions of the generalized Legendre equation,

$$(1 - x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + \left\{n(n+1) - \frac{m^2}{1-x^2}\right\} y = 0,$$

which arises in problems concerned with the determination of potential functions without axial symmetry, in spherical polar co-ordinates. In particular, if m is zero, it is clear that they reduce to the ordinary Legendre functions, of which the former, denoted merely by $P_n(x)$, is a polynomial of degree n, which plays a supremely important part in elementary potential theory. From this as a starting point, it is easy to deduce that if x is a real number numerically less than unity,

$$P_n^m(x) \equiv (1 - x^2)^{m/2}\frac{d^m}{dx^m}[P_n(x)],$$

with a similar formula for $Q_n^m(x)$.

Clearly, if m and n are positive integers, $P_n^m(x)$ vanishes identically for $m > n$; and for $m = n$ it is a constant multiple of $(1 - x^2)^{m/2}$ for any given m. We are thus concerned in tabulation of functions of integral degree only with the case $m \leqslant n$.

In this volume only tables of $P_n^m(x)$ appear, and since

$$P_n^m(-x) \equiv (-1)^{n-m} P_n^m(x),$$

the range $0 < x < 1$ is clearly sufficient. m and n each take all (appropriate) integer values from 1 to 10, and the entries for x are at intervals of 0·001. There are thus 55,000 entries in all, and these are spread over 280 pages. For $n \leqslant 6$ and $m \leqslant 5$, each entry is given correct to eight decimal places; but owing to the magnitude of the functions for larger values of m and n the decimal place accuracy is progressively reduced to four.

Two final pages of Everett coefficients are appended, and by means of these and suitably modified second differences supplied with each functional entry it is possible, following the lines of an example worked out in the introductory pages, to interpolate for intervals of 0·000001 to a remarkably high degree of accuracy.

Eight pages of introductory material provide a very lucid account of the definitions and properties of $P_n^m(x)$, together with an explanation of the methods of calculation of the entries and the checking processes employed. A preface, in Egyptian, at the end, completes the volume, which is admirably bound and clearly printed.

J. H. Pearce

NEW SCHOOL CHEMISTRIES

A New Introduction to Chemistry

By H. L. Heys. Pp. 410. (London, Bombay and Sydney: George G. Harrap and Co., Ltd., 1945.) 6*s*. 6*d*.

Chemistry for Junior Forms

By A. C. Cavell. Pp. viii + 296. (London: Macmillan and Co., Ltd., 1946.) 5*s*.

In the interval since pre-war text-books were written many advances in science have been made. New materials (for example, plastics and penicillin) have become widely available, new processes have established themselves and there has been marked progress in our knowledge of fundamental principles. Writers of elementary text-books are now faced with the problem of incorporating this new information into their books, which must at the same time have regard to examination requirements at the end of the school certificate and higher school certificate courses. These two books are both very suitable for school use and should prove popular among science masters and their pupils.

Mr. Heys' "New Introduction to Chemistry" covers much ground and does it extremely well. It begins with a concise historical sketch leading up to an account of combustion phenomena and continuing with an experimental treatment of the air, water, etc. Theoretical principles are then conveniently introduced before the more descriptive sections, these ending with nearly thirty pages on organic compounds. The author gives special attention to the part played by chemistry in the home and in everyday life. He has not hesitated to give, in the appropriate connexion, simple accounts of emulsions; the treatment of sewage; new applications of catalysis and uses of metals; atmospheric pollution; the products from coal and petroleum, the manufacture of petrol from coal, and other things that young readers really like to know.

Mr. Cavell's "Chemistry for Junior Forms" is written rather more simply, but has the great merit that it does not give the results of the experiments the pupil is asked to perform himself. In fact, it is intended that the experiments should be carried out before the rest of the chapter is read. This procedure will, therefore, appeal to those interested in the heuristic method of teaching science. In this respect it would seem that the author has left just enough for the pupil to discover for himself and having found the answer he is stimulated to attack the next problem.

Both books are well illustrated and have good indexes. Both include suitable revision questions at the end of each chapter. In Mr. Heys' book there is one set of questions for short or oral answers and another for longer written answers.

J. G. F. Druce