If Steinmetz's empirical law ($W = \eta(B_{\max})^k$ ($k \approx 1.6$) is applicable to our experimental data, then there must exist a linear relation between $\log W$ and $\log B_{\max}$; we find such a linear variation, with the approximate error of 1 per cent, between the flux-densities of 1,000 and 12,000 gauss, with the following values for the constants η and k :

	Sheet 1	Sheet 2	Sheet 3
η	0.00076	0.0010	0.0011
k	1.66	1.65	1.59

It was found that Steinmetz's law is not applicable at high and low flux-densities. We have observed, like Brailsford¹, that for sheets 1 and 3, and very high values of $B_{\max}$, the hysteresis loss increases with $B_{\max}$ according to a linear law.

Moreover, it has been shown experimentally that, in accordance with theory, the influence of a superimposed steady unidirectional flux on the hysteresis loss is nil. The measurements were carried out with steel sheet 1 and for three different values of steady flux: 1,500, 4,100 and 11,300 gauss. 4,100 gauss is the value of the flux-density for which magnetic permeability of the material is a maximum. As an example, the experimental data for two loops of very different shape are: loop with steady flux = nil: $B_{\max} = 13,360$ gauss; $W = 6.47 \times 10^8$ erg.cm.⁻²; loop with steady flux = 11,300 gauss: $B_{\max} = 13,320$ gauss; $W = 6.31 \times 10^8$ erg.cm.⁻².

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¹ Brailsford, F., *J. Inst. Elect. Eng.*, **90**, 313 (1943).

Biological Action of Radiations

It has been shown previously¹ that when radiation (for example, X-rays, α -particles, neutrons) is absorbed in aqueous solutions, an indirect chemical effect is produced due to the formation of free radicals and atoms:

In general, this is followed by the recombination:

and by interactions of these radicals with other acceptor substances (S , S_1 , S_2 , ...) present, for example:

This scheme of reactions must exist also in the aqueous systems of biological subjects. From the point of view of the older 'hit' theory², reaction (3) corresponds to a single 'hit', while if, for example,

$$E_1 = S_1, E_2 = S_2, \dots, E_{n-1} = S_{n-1} \quad (4)$$

one gets a successive change of the (starting) substrate (S) by 1, 2, ..., n interactions with, for example, OH radicals, which are equivalent to 1, 2, ..., n 'hits'. It can be shown that—under certain conditions—there is a complete formal analogy between the 'hit' theory, which is described by Poisson's formula, and the mathematical expression for the above system of equations (3, 3.1, ..., 3.n).

If the biologically active substance is represented by the molecule S , the essential interaction corresponds to equation 3 and the theory suggests two important cases:

Case A. The recombination process (2) can be neglected (if the interaction with the acceptor molecules is relatively fast). From the above scheme of reactions, it follows that in this case the action of the radiation (for example, the half 'lethal' dose $D_{1/2}$) depends only on the total number of ions (radicals) produced, independent of the nature and wave-length of the radiation. This case is represented, for example, by the mutations induced by X-rays. Under the assumption—which is almost certainly fulfilled—that apart from the S molecules there are also some other (biologically non-active) molecules present which react with the OH-radicals (thereby exerting a 'protection effect') the chemical theory leads to the equation:

$$S = S_0 \exp. (-\alpha D) \quad (5)$$

where S and S_0 represent the numbers of these molecules at times t and zero respectively; D , the total dose of radiation and:

$$\alpha = \ln 2/D_{1/2} \sim q_s(q_{sm}[S_m])^{-1} \quad (6)$$

where q_s represents the (kinetic) cross-section of the S molecules and the term in brackets can be interpreted as the mean linear region available per one molecule of the (protecting) substrate molecules (S_m).

Case B. Recombination reaction (2) taken into account (if the radicals are present in relatively high concentrations or, for example, the acceptor molecules are relatively non-reactive). It follows that in this case the local concentration and distribution of the radicals is of importance. While for different radiations the number of ions (radicals) produced is approximately the same for equal doses, their local distribution may differ widely. In this case, therefore, the action of the

radiation must depend on the nature of the radiation, as is known to be the case, for example, of the lethal effect on unicellular organisms.

The above scheme leads to the following expression (with certain simplifying assumptions):

$$S = S_0 \exp. \left(- \frac{k_3 D}{k_2 [H]_l} \right) = S_0 \exp. (-\alpha D) \quad (7)$$

where $[H]_l$ represents the local concentration of the radicals in the regions where the actual reaction takes place. As a first approximation $[H]_l$ can be estimated for two important stages of the reaction. (i) In the initial stage, that is, before any appreciable diffusion of the radicals (formed in the tracks of the ionizing particles) has set in. Here one may assume that:

$$[H]_{l(i)} \propto \text{number of ions per unit length of track} \quad (8)$$

This quantity is known experimentally^{3,4} for different radiations. More generally, one has approximately^{5,6} (if V denotes the energy of the radiation):

$$[H]_{l(i)} \propto V^{-1} \text{ (for X-rays)} \quad (9)$$

$$[H]_{l(i)} \propto V^{-1/2} \text{ (for } \alpha\text{-rays)} \quad (9a)$$

(ii) In the advanced stage (but before any appreciable intermixing of the individual ionization tracks has occurred):

$$[H]_{l(ii)} \propto V \quad (10)$$

Taking into account both these stages for the action of, say, X-rays, one obtains for α of equation 7,

$$\alpha = \alpha_{(i)} + \alpha_{(ii)} \propto \frac{k_3}{k_2} \frac{1}{V} \left(\frac{V^2}{a} + 1 \right) \quad (11)$$

where a denotes a proportionality factor. Remembering that $V \propto \lambda^{-1}$ and that for the range (R_e) of the (ionizing) fast electrons one has the relation⁷: $R_e \propto V^2$, one obtains finally from equations 7 and 11

$$D_{1/2} = \ln 2/\alpha \propto \frac{1}{\lambda} \frac{a}{(R_e + a)}, \quad (12)$$

which is formally identical with Glocker's well-known expression.

It is now possible to predict that if sufficiently hard γ -rays or X-rays are used, the dependence on the wave-length should gradually diminish and finally disappear. This can be illustrated by Wyckoff's own figures⁸ for *Bacterium coli*, where the mean lethal dose decreases in the series of decreasing wave-length as: 8.4, 6.67, 4.65, $4.20 \times 10^4 r$, and in order to achieve the same effect with α -rays, $24 \times 10^4 r$ are required. On the other hand, while it is known that the doses of radiation for the production of mutations are independent of the wave-length in the case of γ - and X-rays⁹ a much higher dose of neutrons is required to produce the same effect¹⁰. This follows directly from the above theory, as the recoil protons produce a relatively high local ion (radical) density leading to increased recombination.

A full account will be published elsewhere which will include also a discussion of 'multiple hit effects', where the situation is somewhat different¹¹.

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¹ Weiss, *Nature*, **153**, 748 (1944).

² cf. Mayneord, *Proc. Roy. Soc. A*, **146**, 867 (1934).

³ cf. Gray and Read, *Brit. J. Radiol.*, **15**, 72 (1942).

⁴ Lea, *Proc. Camb. Phil. Soc.*, **30**, 80 (1934); *Brit. J. Radiol.*, **16**, 338 (1943).

⁵ cf. Bethe and Bacher, *Rev. Mod. Phys.*, **8** (1936).

⁶ Bagge, *Ann. Phys.*, **30**, 72 (1937).

⁷ Glocker, *Z. Phys.*, **77**, 653 (1932).

⁸ Wyckoff, *J. Exp. Med.*, **51**, 921 (1930); **52**, 435, 769 (1930).

⁹ cf. Schrödinger, "What is Life?" (Cambridge, 1944).

¹⁰ Timoféeff-Ressovsky and Zimmer, *Naturwiss.*, **26**, 362 (1938).

¹¹ cf. Hevesy, *Rev. Mod. Phys.*, **17**, 102 (1945).

Effect of Container Walls on Packing Density of Particles

IN recent correspondence¹ in *Nature* a general theorem on the ordered packing of equal spheres is cited as implying that "the overall voidage of any one of the four possible ordered arrangements of equal spheres packed into a container is independent of the relative size of the container provided its size and shape are such that it will contain an integral number of 'unit' cells of the particular ordered arrangement and in all other cases an ordered packing cannot be obtained". This is due to a misconception of the 'unit' cell, which in the first three out of the four ordered packings, namely, a sphere having 12, 10, 8 or 6 neighbours, really contains fractions of a sphere totalling to one, and it is only in the fourth case that the unit cell, being a simple cube, contains a whole sphere.

Furthermore, the inescapable higher voidage adjacent to the container walls as compared to the interior of the container has to be taken into account², and its contribution to the overall voidage has to be allowed for; unless it happens to be negligible, as with a container of very large size compared to the particle size of the packed material. Characteristic voidage for a compacted packing of a sized material is then found to depend more on the particle shape^{3,4,5} than on the particle size. This increases with increased deviation of the irregular particles from a spherical shape, unless conditions exist to

promote bridge formation, such as, for example, extremely fine particles or extremely loose packing¹.

If it is assumed that under given conditions of packing a specific ordered or disordered arrangement of particles of a sized material results, depending only on the method of packing and particle shape, then this arrangement may be taken to exist in the central regions of any container thus packed. Now supposing in a very large container assumed to be packed in this manner an imaginary plane is inserted, then this plane will cut through a given number of particles on the average. If this plane is now to be regarded as a constricting wall of a container, the number of particles cut through by the plane will have to be eliminated, and the extra voidage thus caused considered as that due to the container wall, provided it is assumed that the container walls have no influence on the detailed arrangement of packing; but only act as a limiting boundary of the general packing arrangement. It can then be shown that

$$d_n = \left(\frac{n-1}{n}\right)^3 \cdot d_0 \quad (1)$$

where d_n is the overall packing density in a cubic container having sides n times particle diameter of the sized material (say, $2nr$) and d_0 is the characteristic packing density of the material in the interior of the container, which may be regarded as specific to the material and the mode of packing; n will, of course, have to be sufficiently large to give a definite physical meaning to d_0 .

Equation (1) would apply equally well to a cylindrical container the diameter and height of which are each equal to $2nr$. It may be further generalized to apply to right-cylinders of unequal dimensions or rectangular containers as follows:

$$d_n = \left(\frac{n_1-1}{n_1}\right) \left(\frac{n_2-1}{n_2}\right) \left(\frac{n_3-1}{n_3}\right) d_0 \quad (2)$$

where n_1 , n_2 and n_3 represent the dimensions of the container in three orthogonal directions in terms of particle diameter.

Equations (1) and (2) may be further shown to hold for a material having more than one component size, in which each small-sized fraction is present in sufficient proportion to fill up the voidages produced by the next larger-sized fraction. In this case n 's will be taken in terms of the smallest-sized fraction. For a similar reason, even in the case of sized material, n 's may be taken more appropriately in terms of smaller-size particles.

Applying equation (1) to the data given by Brown and Hawksley¹ for packing two sizes of graded coal in a 10 in. cube container, we find that, if $d_0 = 0.60$, the voidages are 0.48 and 0.42, as compared with the experimental values of 0.47 and 0.43 respectively.

Container walls, however, definitely influence the distribution of neighbouring particles and not merely interrupt this distribution as implied above. Assuming that this influence of the walls on the spatial distribution of particles is confined to the first layer of particles immediately next to the boundary, equation (1) may be modified as follows:

$$d_n = \left(\frac{n-1}{n}\right)^3 d_0 + \left[1 - \left(\frac{n-2}{n}\right)^3\right] \Delta d \quad (3)$$

where Δd represents the deviation of packing density at the boundary layer from what it would have been had the walls been assumed to have no effect on the general distribution of particles, represented by a packing density d_0 .

This equation, when applied to Brown and Hawksley's data cited above, gives $d_0 = 0.58$ and $\Delta d = 0.11$.

Sufficient experimental data are not available in the literature to establish finally the general applicability of these equations, but work dealing with experimental aspects is in progress.

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¹Brown, R. L., and Hawksley, P. G. W., *Nature*, **156**, 421 (1945).

²Anderegg, F. O., *J. Ind. Eng. Chem.*, **23**, 1062 (1931).

³Fraser, H. J., *J. Geology (Chicago)*, **43**, 936 (1935).

⁴Wadell, H., *J. Geology (Chicago)*, **43**, 276 (1935).

⁵Furnas, C. G., *J. Ind. Eng. Chem.*, **23**, 1052 (1931).

⁶Meldau, R., and Stach, E., *J. Ind. Fuels*, **7**, 336 (1934).

Dr. Verman and Dr. Banerjee rightly direct attention to a point we overlooked in a previous communication¹ on containers which will fit the unit cells of regular packings of equal spheres. The spheres in the unit cell are sliced and can be reshuffled to give whole spheres in a given container only for a cubic packing. For the other three ordered arrangements a slight adjustment of the spacing of the walls is needed to accommodate the whole spheres.

In connexion with the equations proposed for assessing wall effect, confirmatory evidence may be cited from experiments we have done on the packing of spheres ($\frac{1}{8}$ in. ball bearings) and of cylindrical disks (diameter $\frac{1}{8}$ in., height $\frac{1}{8}$ in.) in cubical metal containers 1-5 in. in size. The packing obtained by pouring the particles rapidly into the container is referred to as loose; that obtained on simultaneously

shaking the container in a horizontal plane is referred to as a tight packing. The containers were overfilled and the surplus particles were swept off with a straight edge. There is some difficulty in defining the top surface in this way for the smaller containers, and the convention that no particle should project above the top of the container was adopted. Thus the inherent error varies as $(1/n)$, where n is the ratio of container to particle size. Ten repeat measurements were made for relative container dimensions of $n = 4-24$. The value of n for disks was found from the equivalent spherical diameter.

Plotting the results in the form y against x' where

$$y = d_n \left(\frac{n}{n-1}\right)^3; \quad x' = \left(\frac{n}{n-1}\right)^3 - \left(\frac{n-2}{n-1}\right)^3 \quad (A)$$

and for which equation 3 proposed by Dr. Verman and Dr. Banerjee suggests the linear relation

$$y = d_0 + x' \Delta d \quad (B)$$

proved to give a slight curve. This was thought to arise from the perturbing influence of the walls extending beyond the immediate adjacent layer of particles. On the other hand, substantial straight-line plots were obtained by plotting y against x where

$$x = \left(\frac{n}{n-1}\right)^3 - \left(\frac{n-3}{n-1}\right)^3 \quad (C)$$

This is shown in the accompanying graph, where the scatter of replicate measurements is plus or minus twice the standard deviation of the average.

It will be evident from the graph that a linear relation of the form of equation B is statistically valid for both loose and tight packings of equal spheres; in both cases Δd is approximately 0.28; d_0 is 0.57 for loose and 0.58 for tight packings. The data are not sufficient to show whether Δd is the same for loose and tight arrangements.

With non-spherical particles, the disks yield the same conclusion, with Δd approximately 0.28, $d_0 = 0.60$ for tight and 0.54 for loose packings. It will be noted that the results for spheres lie between those for disks.

It may also be of interest to give the results for irregularly shaped coal particles sized $0 \times \frac{1}{8}$ in., $\frac{1}{8}$ in. $\times \frac{1}{8}$ in., $\frac{1}{8}$ in. $\times \frac{1}{8}$ in. $\times \frac{1}{8}$ in. $\times \frac{1}{8}$ in. sq. mesh ($n = \infty, 80, 40, 20$ respectively in a 10-in. cube). Here the accuracy suffices only to show that Δd is much smaller than with disks, while d_0 is 0.59. Further work is needed to test the validity of equation 3 with coal particles.

It would appear that d_0 (or the interior packing) within the range of our experiments has an average meaning down to $n = 4$, which corresponds to about sixty particles.

In conclusion, it may be noted that Mr. Leftwich's calculations² provide a good test of the equation proposed by Verman and Banerjee for the geometrical effect of walls around ordered packings of equal spheres. With slight discrepancies the calculations follow the equation

$$d_n = \left(\frac{n-\frac{1}{2}}{n}\right)^3 d_0 \quad (D)$$

with $d_0 = 0.7405$. The modification of $(n-1)$ to $(n-\frac{1}{2})$ arises from the choice in the calculations of five planes tangent to a sphere layer (and therefore 'cutting' on one side only); equation D is valid for six such tangent planes, and the discrepancies presumably arise from a small unfilled region at the top of the box.

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¹*Nature*, **156**, 421 (1945).

²*Nature*, **156**, 753 (1945).