A Definition of Plasticity

THE word 'plasticity', although one which has attained a state almost of ubiquity in several branches of applied physics and industrial chemistry, has, according to Scott Blair[1], never been given a quantitative and dimensional definition. In the technical literature a group of quantitative 'plasticity numbers' have taken their place. Such expressions as the Williams plasticity number, or the Goodrich plasticity number (both used in rubber technology), have no quantitative meaning other than that of the reading of a particular type of testing equipment when used in a particular manner, and as such cannot be expressed in C.G.S. units.

An excellent qualitative definition of plasticity has been given by Wilson[2] and is quoted by Scott Blair. Plasticity is defined as "that property which enables a material to be deformed continuously and permanently without rupture during the application of a force that exceeds the yield value of the material". As it stands, this definition does not permit of a quantitative interpretation, but it is not difficult to make it quantitative.

If we denote the stress acting by τ and the strain produced by σ then we may write generally for the behaviour of a solid

$$\tau = f(\sigma) \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The stress is a function only of the strain. For a fluid body generally:

$$\tau = f\left(\frac{d\sigma}{dt}\right) \quad . \quad . \quad . \quad . \quad . \quad (2)$$

For a plastic body:

$$\tau = f\left(\sigma, \frac{d\sigma}{dt}\right).$$

This expression may be perhaps more simply expressed by the fractional differential equation

$$\tau = f\left(\frac{d^a\sigma}{dt^a}\right) \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where the exponent a, which lies between 0 and 1, varies with the magnitude of the stress. The qualitative definition of Wilson above implies that for a plastic body the value of the exponent a changes fairly rapidly from 0 to 1 as the stress passes the yield value. We may therefore define plasticity quantitatively as the reciprocal of the relative stress-change required to change a from zero to unity. As, however, both these limiting values may be approached asymptotically, it might be better to define plasticity as *the reciprocal of twice the relative stress change required to change the exponent* a *from* 1/4 *to* 3/4.

To measure the plasticity of a material in terms of this definition, we divide the material into a number of samples. To each sample a different stress τ is applied and the strain σ measured as a function of the time. Curves of $\log \sigma$ versus $\log t$ are then drawn. Up to the yield point, these curves will be straight lines parallel to the $\log t$ axis, but at higher stresses positive slopes $\left\{\frac{d \log \sigma}{d \log t}\right\}$ will be obtained. As both $\log \sigma$ and $\log t$ are dimensionless quantities, the slopes will also be dimensionless and the stress required to attain a given value of the slope will have the dimensions $[ML^{-1}T^{-2}]$.

As illustrated by the work of Andrade and Chambers[3], the expression $d \log \sigma / d \log t$ may not always be independent of t (the time elapsing since applying the stress τ). We may, however, characterize each curve of $\log \sigma$ versus $\log t$ at constant τ by the fixed initial value of the slope, namely:

$$\left(\frac{d \log \sigma}{d \log t}\right)_0.$$

We next proceed to plot this quantity against the stress τ. Let $\tau_{1/4}$ be the value of τ required to give a slope of 1/4, and let $\tau_{3/4}$ be that required to give a slope of 3/4. We then give as the quantitative definition of plasticity:

$$\text{Plasticity} = \frac{\tau_{3/4} + \tau_{1/4}}{\tau_{3/4} - \tau_{1/4}} \quad . \quad . \quad . \quad (4)$$

In terms of this definition, plasticity becomes a dimensionless quantity having the same magnitude in all units of measurement.

In illustration of the definition given above, a few simple examples will now be considered. In a Bingham system, that is, one for which the rate of flow follows the law;

$$\frac{d\sigma}{dt} = 0 \qquad \tau < f$$
$$\frac{d\sigma}{dt} = \frac{1}{\eta}(\tau - f) \qquad \tau > f, \ \eta \text{ a constant.}$$
$$\tau_{1/4} = \tau_{3/4} = f$$

and the plasticity in terms of equation (4) is infinite.

In a system which breaks down fairly sharply from that of an elastic solid to one which Reiner[4] would term a St. Venant plastic, $\tau_{1/4}$ and $\tau_{3/4}$ are both finite and $\tau_{3/4}$ is slightly larger than $\tau_{1/4}$. The plasticity of such a system is (by equation 4) a large positive number, being the larger the more rapid the change-over from elastic to plastic behaviour.

For both brittle solids and fluids (either Newtonian or non-Newtonian) $\tau_{1/4}$ and $\tau_{3/4}$ are both purely imaginary and the plasticity as defined by equation (4) is indeterminate.

For systems undergoing crystalline glide with associated shear hardening, the rate of flow $d\sigma/dt$ and constant stress τ decreases with time and ultimately becomes zero. For such systems the value of the exponent a is always less than unity. Should a in addition never attain the value 0·75, $\tau_{3/4} \rightarrow \infty$ while $\tau_{1/4}$ remains finite. For such systems the plasticity, according to the definition suggested in this letter, is unity.

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[1] Scott Blair, G. W., "A Survey of General and Applied Rheology" (London: Pitman, 1944).
[2] Wilson, H., "Ceramics, Clay Technology" (New York: McGraw-Hill, 1927), 55.
[3] Andrade, E. N. da C., and Chambers, B., *Proc. Roy. Soc.*, A, **138**, 348 (1932).
[4] Reiner, M.. *J. Sci. Instruments*, **22**, 127 (1945).

Need for the Development of Tropical Ecological Stations

WE emphatically agree with most of the recent letter from Dr. Chapman and others[1] on the need for the development of tropical ecological studies. Such studies are essential to the work of the practical applied scientist—the agriculturalist or forester—and also in the field of pure ecology. There are, however, a number of points which seem to be wrongly stressed, or left unmentioned. Dr. Chapman *et al.* themselves state that theoretical ecology is to-day to some extent based on unbalanced data, drawn from only a part of the world's vegetation. It is possible that much modification may be necessary to this theory as the details of vegetation in regions with predominantly phanerophytic floras become known.

One point must be questioned. Dr. Chapman and his collaborators state that the tropics require synecology before all else; and further, that factors other than mere floristics should be regarded as the main "Characters determining the aspect and form of the community".

We submit that for any rational understanding of vegetation—and that is the ecologist's primary concern—full investigation must be made into all the main determinative factors of vegetational distribution and relations. These are species distribution, climate, geology and soils, water relations and drainage, topography and biotic effects. Only when *all* are considered together is there any hope of reaching a true understanding of the vegetation. To decry the first on the grounds that it is 'difficult' is scarcely reasonable.

The real reason why such a course is promulgated is that in most tropical regions, certainly throughout Africa, our present knowledge of the flora is limited. In Europe, with the flora at a high level of taxonomic resolution, ecological studies are possible at any level, and any particular aspect can be stressed. Further taxonomy must go hand-in-hand with ecological studies, and the latter must not be attempted unless accompanied by the former. To suggest that this limiting factor of taxonomic resolution can be by-passed by concentration on other more abstract and intangible criteria is merely to court disaster.

We have recently attempted to carry out synecological work in Nigeria; in fact, the mandate of duty for one of us is largely in agreement with Dr. Chapman's suggestion that one or more officers should be specifically directed to write up descriptions of various types of vegetation and any ecological conclusions that may emerge. Our progress in these studies has been entirely dependent upon a fuller and more critical taxonomic acquaintance with the West African flora. This applies as much to the wet forest regions as to the drier savannahs. One of us can recall the failure of early attempts to understand dry-zone vegetation, in the light of his university training in ecology. The importance of floristic studies was under-emphasized in this training and not over-emphasized, as the writers of the letter suggest. It was only when the problems of floristics, both distributional and taxonomic, were considered, that any real understanding of the vegetation *in terms of its causal agents* was achieved. We do not, we trust, as former pupils of two of the signatories, lack appreciation of the need for studying other aspects of the vegetation.

In conclusion, may we suggest that, while wholeheartedly supporting the plea for more tropical ecological work under a more widely based and competent directive, there should be, nevertheless, the fullest possible co-operation between the field ecologist and the systematists working on the partially resolved tropical floras.

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[1] See *Nature*, **157**, 377 (1946).

Literature for the Hong Kong Fisheries Research Station

As a result of Japanese aggression in Hong Kong, the entire library of the Fisheries Research Station and the private libraries of myself and of Mr. S. Y. Lin have been lost. We were particularly unfortunate in that a Japanese bomb completely demolished the field station situated at Aberdeen, Hong Kong, and with it all the specimens and records contained therein; others housed in the science building at the University were also lost as the building was gutted by fire.

Among records missing is the list of persons and institutions with whom publications were exchanged for the *Hong Kong Naturalist* and *Supplement* and the *Journal of the Hong Kong Fisheries Research Station*. The Fisheries Department, with temporary offices in the G.P.O. Building, Hong Kong, would be grateful if any individuals, or institutions, who would like to make or renew contacts, or to present literature, would write to the officer in charge.

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Dec. 3.