A New Test for 2 × 2 Tables

IF an experiment yields results in the form of a 2 × 2 table:

	P	not-P	Total
A	a	c	m
B	b	d	n
Total	r	s	N

where m and n have been fixed in advance, a test for deciding whether there is evidence of association between the attributes A and B and the mutually exclusive and exhaustive attributes P and not-P has been given by Fisher[1]. This test, on the null hypothesis of no association, associates the results with a probability $m!n!r!s!/N!a!b!c!d!$.

It has been usual in the past to apply this test, or some approximation to it[2,3], in cases where we can assume that the probability p_1 that A has P and the probability p_2 that B has P are both constant. The hypothesis tested then becomes $H(p) \equiv p_1 = p_2 = p$.

It is, however, possible to construct a more powerful test of the hypothesis $H(p)$ on the data given. The table above can be represented geometrically by the point (a,b) in a plane lattice diagram of points with integer co-ordinates. Since m and n are fixed in advance, all possible results will then be represented by points of the lattice lying in a rectangle bounded by the x and y axes and the lines $x = m$ and $y = n$. The hypothesis $H(p)$ assigns a 'weight'

$$W(a,b,p) = (m!n!/a!b!c!d!)p^r(1-p)^s$$

to the point (a,b). To obtain a valid test of $H(p)$ on significance level α, we have only to choose a region R in the rectangle such that

$$\underset{0 \leqslant p \leqslant 1}{\text{Max}} \sum_R W(a,b,p) \leqslant \alpha.$$

This validity condition does not determine R uniquely. To obtain a reasonable test, we must require that R should consist of as many points as possible, and should lie away from that diagonal of the rectangle which passes through the origin. Formulated mathematically, these latter requirements mean that the complement of R must in a certain sense be convex, symmetrical and minimal.

For example, when $m = n = 3$, and $\alpha = 1/32$, R consists of the two points (3,0), (0,3). The corresponding level of significance with Fisher's test is 1/10. Thus the new test is more powerful than Fisher's.

The computation involved in making tables for the new test is heavier than with Fisher's test. But once prepared, the tables for the new test are on the whole more convenient in use.

The relationship between Fisher's test and the new test is similar to that between Pitman's 'exact' test for identity of two distributions and the corresponding t-test. For large values of a, b, c and d, the difference between the two tests becomes small. For large m,n and small a,b, there is an analogue of the new procedure, just as there is an analogue of Fisher's[4].

This work has been carried out as part of the programme of the Ministry of Supply. Full details, with tables, will be published elsewhere.

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Crystal Structure of Barium Titanate

RECENTLY Rooksby[1] and Megaw[2] have reported a tetragonal structure for barium titanate. This is of interest to us because we also have obtained X-ray powder photographs of barium titanate, from which we have established a tetragonal structure. In addition we have obtained photographs of a lattice which at first was regarded as near cubic, though in view of our subsequent findings, it appears more correct to regard it as being near tetragonal. However, the reversible temperature change of structure from tetragonal to cubic reported by Miss Megaw[2] raises the question as to whether this cubic structure might not exist at ordinary temperatures, as a result of heat treatment, or method of preparation, or presence of impurities.

The energy difference between the structures may not be large, but the change from tetragonal to cubic would appear to involve a rearrangement of the ions if the cubic structure is to be regarded as of the ideal $G5$ type, as hitherto assumed[3]. The change in structure may be connected with a very slight change in the Ti—O bond-length.

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Particle Shape

THE recent letters in *Nature* by Messrs. Whittaker and Tomkeieff[1] make no mention of the extensive investigations of particle shape carried out in connexion with road aggregates. A bibliography of the subject is given in my first paper[2].

In my work at the Road Research Laboratory[2,3], shape was defined in terms of the principal dimensions of the particle, which correspond roughly to its length, breadth and thickness. The ratios of length to breadth and thickness to breadth are taken as a measure of the degree of elongation and flakiness of the particle. By selecting arbitrary limits, it is possible to determine the amount of 'elongated' and 'flaky' particles and to express these in percentage by weight as indexes of shape. This assumes a more or less regular distribution of particle shapes in such materials, which has been found to be the case in practice[2,3]. This is the basis of the method adopted in British Standard 812: 1943, "Sampling and Testing of Mineral Aggregates Sands and Fillers" (Method 2).

This method, of course, only gives a partial definition of particle shape. An obvious omission is the degree of roundness of the particles, which is independent of the 'shape' as defined above. Another shape characteristic is the angularity of crushed particles. Particles classified on length and thickness can be further classified according to these characteristics. The shape characteristics of interest to any industry must be determined to suit the particular case.

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