LETTERS TO THE EDITORS

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Dirac's Equation for the Neutron and Proton

IN the attempt to discover a place for Dirac's equation in the geometry and metric of the physical world, it has been proposed to make use of Weyl's concept of gauging. It was thus proposed to bring about the union of gravitation and electromagnetism by Kaluza's suggestion of the adoption of a five-dimensional continuum leaving the idea of gauging free to bring about the union of the quantum theory with these branches of physics.

In order to carry out this idea, it was found necessary to introduce the concept of matrix-length and to apply a parallel displacement to it. A gauging factor, ψ, was introduced which was identified with the ψ of Dirac's equation. The result of the argument was the equation

$$\gamma^{\mu}\frac{\partial\psi}{\partial x^{\mu}} = 0,$$

as the condition that there should be no change in length in a parallel displacement. In this equation the matrices (γ^{μ}) are five in number and are readily related to the familiar Dirac matrices. The equation can be regarded as a gauging equation and is identical with Dirac's equation[1].

In the original work the equation was more general and can be expressed in the form :

$$\gamma^{\mu}\frac{\partial\psi}{\partial x^{\mu}} = H_{\mu}\gamma^{\mu}\psi.$$

This is the result of a close analogy with the idea of Weyl that a change of length occurs with a parallel displacement, and in place of the electromagnetic potential (φ_{μ}), the operator (H_{μ}) occurs. It appeared from the identity of the equation, which resulted from placing $H_{\mu} = 0$, with Dirac's equation that the purpose of incorporating Dirac's equation in a natural way into physical theory was achieved by this simple condition. But the development of the theory of the nuclear field has shown that it bears to the five-dimensional framework a relation similar to that which exists between the electromagnetic theory and gravitation[2]. Thus it would be natural to drop the simple equation $H_{\mu} = 0$ when a nuclear field exists. If we can decide upon the correct expression for H_{μ} in this case, the result should lead to Dirac's equation for the neutron and proton.

Following Weyl, it would be expected that H_{μ} depended linearly upon the potential components, usually denoted by U_{μ}. But in the five-dimensional theory these are the components, ($T_{\mu\nu}$), of the tensor of the nuclear field in which the suffix 5 appears. Thus it is in the interest of generality to express H_{μ} linearly in terms of the components, $T_{\mu\nu}$.

The simplest operator form is that which results from combining the matrices (γ^{μ}) with ($T_{\mu\nu}$) in order to produce a covariant quantity H_{μ}. At the same time it is necessary to include the matrices $\tau = \begin{pmatrix}00\\10\end{pmatrix}$ and $\tau^* = \begin{pmatrix}01\\00\end{pmatrix}$ to account for the neutron-proton interchange. Thus the simplest form is $H_{\mu} = g\gamma^{\nu}(T_{\mu\nu}\tau + T^*_{\mu\nu}\tau^*)$, where g is a constant, and where as usual in the field theory we combine $T_{\mu\nu}\tau$ and its conjugate complex quantity. Thus the proposed quantum equation for the neutron and proton is

$$\gamma^{\mu}\frac{\partial\psi}{\partial x^{\mu}} = g\gamma^{\nu}\gamma^{\mu}(T_{\mu\nu}\tau + T^*_{\mu\nu}\tau^*)\,\psi.$$

This equation appears in a paper by H. J. Bhabha[3], in which he develops equations for the nuclear field by the Hamiltonian method. A characteristic feature of the form of the equation developed here is the appearance of one constant, g, only. In other presentations the field is characterized by the appearance of two constants, g_1 and g_2. It would appear from this equation that $g_1/g_2 = 2\pi M_0\,c/h$, where M_0 is the mass of the particle emitted in the neutron-proton interchange. This agrees with the five-dimensional formulation of the nuclear theory[4].

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[1] Flint, *Proc. Roy. Soc.*, A, **150**, 432 (1935).
[2] Flint, *Phil. Mag.*, vii, **29**, 330 (1940).
[3] Bhabha, *Proc. Roy. Soc.*, A, **166**, 520 (1938).
[4] Flint, *Phil. Mag.*, 7, **33**, 369 (1942).

Coloured Haloes Surrounding Inclusions of Monazite in Quartz

QUARTZ crystals with zonal colour from the Trans-baikal region were found with inclusions of monazite crystals (diameter 0·1–1·0 mm.) around which the quartz was of an intense smoky colour. The coloration, more intense at the centre, shaded off gradually towards the periphery of the halo. Although the boundaries of the halo were not sharp, yet they were sufficiently distinct to enable measurements of the halo to be made (Fig. 1). Haloes of all dimensions from 0·5 mm. to 5–6 mm. diameter can be found in the same quartz specimen.

The formation of these spherical haloes can be attributed to the corresponding radioactive emanations (β or γ). Calculations have shown that haloes of such dimensions and with sufficiently well-marked boundaries could be produced by the action on quartz of hard β-rays (in quartz $\mu \simeq 10$ cm.$^{-1}$) spreading

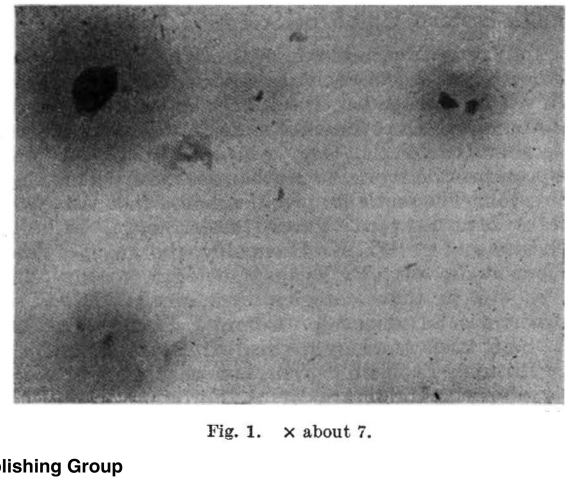

Fig. 1. × about 7.