

The stability of the first system by no means implies the stability of the second system. It is clear, then, that fixing a variable may render the rest of a system unstable.

As a numerical illustration, the system

$$\left. \begin{aligned} \dot{x}_1 &= 6x_1 + 5x_2 - 10x_3 \\ \dot{x}_2 &= -4x_1 - 3x_2 - x_3 \\ \dot{x}_3 &= 4x_1 + 2x_2 - 6x_3 \end{aligned} \right\}$$

leads to the equation

$$\lambda^3 + 3\lambda^2 + 26\lambda + 60 = 0;$$

and this has roots $-2.44, -0.28 \pm 4.95i$, where $i = \sqrt{-1}$. The real parts being all negative, the system is stable. But if we fix x_3 , we have a system with determinant

$$\begin{vmatrix} 6 & 5 \\ -4 & -3 \end{vmatrix},$$

and as the roots are now $+1$ and $+2$, the system is unstable.

We can, however, go further than this. Since the sum of the roots is equal to the sum of the elements in the main diagonal, Σa_{ii} , any change making this less negative will tend to make the system less stable—other things being equal (the argument here is admittedly imprecise). So the fixing of x_n would be particularly likely to lead to instability if a_{nn} was large and negative. We can identify such variables without difficulty; for, as they behave in accordance with the equation

$$\frac{dx}{dt} = \zeta + ax,$$

where ζ is independent of x , but changes with time, while a is large and negative, such a variable (x) will always have the properties that (1) it always moves towards $-\zeta/a$, (2) it moves towards $-\zeta/a$ quickly, (3) as $-\zeta/a$ has a as denominator it will be small, and therefore the fluctuations of x will be small.

It is concluded, therefore, that: (1) To fix a sociological or economic variable by order carries some danger of rendering the system, or parts of it, unstable (the latter being shown by the subsequent development of various 'vicious circles'). (2) The type of variable more particularly dangerous from this point of view is one which, under free conditions, changes value at high speed, and, by these quick changes affecting the other variables, fluctuates only through a narrow range.

Not being an economist I cannot give detailed instances, but I have little doubt that some could be provided.

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Vibrations in Telegraph Wires

WALKING near Winchcomb on December 27, we noticed a most curious phenomenon. We were in a slight valley, and the road was crossed by a line of telegraph wires, there being two wires some distance apart. It was about 10.30 a.m. and the sun, shining on the wires, was just beginning to thaw the layers of frost on them. The temperature of the air must have been round about freezing-point, and there was a slight mist. But we felt no breath of wind whatsoever, nor did the dead leaves of trees very near the wires indicate any wind.

But when we arrived on the scene, the wires between two of the poles were vibrating in a very odd manner. The effect was startling as there was no other noise in the neighbourhood, and there appeared to be no cause whatsoever. The wires vibrated with about 4 nodes in them, and with amplitude roughly half an inch; frequency approximately 10 a second. The vibrations went through maxima at irregular intervals, about 2 seconds apart, and we think both wires vibrated independently. The layer of frost was slowly coming off the wires, though some pieces circled round for several minutes. The cause of the vibrations seemed to be the section of the wire just being thawed, and though the vibrations were, of course, transmitted to the neighbouring sections, all the rest of the wire was quite normal. The poles on either side shook considerably, and the effect of putting one's head against them was very like that of being in a bus. But there was no earthquake, all other objects, trees or fences, being quite still. The vibrations were decreasing in amplitude about 10 minutes after our arrival, and when we returned after half an hour, they had ceased completely.

We feel sure an explanation of this phenomenon must involve the layer of frost round the wire. The only suggestion we can make is that when the layer was formed, considerable strains were set up in the wire. When it thawed, the strains were relieved suddenly in various small parts of the wire, each time giving the vibrations a small impulse. There must then be enough such impulses to maintain continuous vibrations. But this theory does not appear very plausible to us. Also we do not think electrical impulses from heavy currents could possibly account for the effect.

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WITH reference to the above communication, we offer the following explanation.

This is a case of self-excited vibration. It would appear that the cross-section of the wire was non-circular due to the secretion of an ice-layer. The cross-section will resemble a short icicle.

If the wire achieves a small downward velocity, and a very slight wind be blowing, aerodynamic reasoning indicates that a force in a downward direction may result. Thus the motion continues, until the elastic forces in the wire stop it. There being now no downward motion, there is no downward force, and the wire commences to rise, in which motion the wind again helps. Thus large vibrations may be set up.

The effect, commonly known as 'galloping', occurs moderately frequently in cold climates; but is rare in temperate zones.

Further information is given in 'Mechanical Vibrations', by Den Hartog, p. 343; and the fiftieth James Forrest Lecture delivered by Prof. C. E. Inglis at the Institution of Civil Engineers in 1944.

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