non-equilibrium state, over the whole surface of given values of the single-value integrals of the motion. The relaxation time is defined as the time of this mixing, that is, the time during which a mixing with such a degree of uniformity is reached as corresponds to the type of the control macroscopic measurement, that is, to the accuracy of the experiment checking the establishment of equilibrium. The relaxation time depends, in general, on the type of the original fluctuation. The largest value of the relaxation time can be defined as the time of mixing of an initial region with a volume A^n, when $A \sim h$.

It turns out that the relaxation time possesses over a very wide range the property of being insensitive with respect to the size of the initial region A^n, tending, however, to infinity in the limit $A \rightarrow 0$, corresponding to a transition to the classical mechanics. From this insensitivity it can be concluded that with increase of the fluctuation the relaxation time increases very slowly, tending rapidly to the limit, corresponding to the minimum value of the region A^n (for $A \sim h$). In the case of an ideal gas, this limiting value of the relaxation time with respect to the velocities is given by the formula :

$$t = \frac{\frac{3}{2}\tau}{\ln \lambda/r_0} \left\{ \ln \frac{2\pi}{(\Delta p/p_0)} \right\},$$

where τ and λ are the duration and length of the mean free path, r_0 is the radius of a molecule, $p_0 = \sqrt{mkT}$, $\Delta p = A/L$, L is the linear dimensions of the system, and $A \sim h$; it thus proves to be of the order of a few τ. In a transition to the classical case, $A \rightarrow 0$ and $t \rightarrow \infty$. Under the condition $A \sim h$, the relaxation time can, however, depend on the choice of the initial region of this size. Since the space of 'initial regions', that is, of the results of the most complete experiments, is compact, it can be shown that the relaxation time for different initial regions of the same minimum size A^n possesses an upper boundary.

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[1] Nopf, *Ber. Ann. Wiss. Leipzig*, **3** (1932).

The Four-Colour Problem

THE correctness of the statement that any plane map may always be tinted with four colours in such a way that two areas meeting on the same boundary never have the same colour has always been admitted since Möbius mentioned it in 1840, although no mathematical proof of this theorem has yet been firmly established. I think I have found a rigorous and general proof of it. First, I have a theoretical proof of the common enunciation ; secondly, I suggest a practical method of colouring. The complete paper is being submitted for publication elsewhere.

(1) Any plane map can be represented schematically as follows. Each area is figured by a point, and every contact between two contiguous areas is represented by a line connecting the figuring points. It is not difficult to show that any map, however complicated, can be reduced to a system of connected triangles externally limited by a unique triangle.

Then, using the method of general induction, I have shown that if it is always possible to colour a net of n vertices by means of four tints, the property remains true for a net of $n + 1$ vertices. In order to make this clear, I have used a theorem which is enunciated as follows : If a net R consisting entirely of triangles and of one—and one only—quadrilateral is colourable, then the net R^1 obtained by adding one supplementary diagonal in the quadrilateral is also colourable.

The evidence that a simple net consisting of a single triangle is colourable leads by induction to the complete demonstration.

(2) The foregoing statement is the basis on which the practical method of colouring rests.

A net already coloured with the four tints, 1, 2, 3, 4, might be coloured otherwise in black and white, making, for example, 1 and 2 white, and 3 and 4 black, in such a way that every polygonal black or white chain is open or when closed involves always an even number of vertices. The reciprocal proposition is true.

On the other hand, it is always possible to classify the vertices of any given net to be coloured in three groups, α, A, B, with two exceptional vertices, and to number the vertices: 1, 2, 3, . . . n, during the process of classification.

By definition, a vertex belonging to group α is directly connected with two and only two vertices, the number of either being lower than its own number. In the same way, a vertex A is connected with three, and a vertex B with four vertices. Nevertheless, it might happen that a vertex is connected with five preceding vertices, belonging then to a new type C. In this case, owing to a definite diagonal mutation of chosen sides of the net, such a vertex enters the group B.

All vertices being numbered and classified in types α, A, B, excepting the two vertices numbered 1 and 2, a general rule can be given for marking the vertices of the net in black and white in such way as to avoid any closed chain, black or white, involving an odd number of vertices.

After that the net can be coloured in the four tints 1, 2, 3, 4.

If any alteration has been made owing to eventual diagonal mutations, it is possible to return to the previous structure of the net, basing the necessary modifications of marking and colouring on the fundamental theorem enunciated in (1).

Bibliography. A short account of the history and bibliography is given in Rouse Ball's "Mathematical Recreations and Essays" (London: Macmillan and Co., Ltd., 1905, pp. 51–54), from which it appears that the problem was mentioned by Möbius in 1840 ; Francis Guthrie communicated it to De Morgan about 1850 ; although familiar to practical map-makers, Cayley redirected attention to it in 1878, but did not know of any rigorous proof of it ; in 1880 Tait published a solution (*Proc. Roy. Soc. Edinburgh*, **10**, 729 ; 1880) but it would seem to involve a fallacy (see J. Peterson, of Copenhagen: *L'Intermédiaire des Mathematiciens*, **5**, 225 ; 1898 ; **6**, 36 ; 1899).

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