differently oriented uniaxial crystal plates ; a problem first attacked by Fresnel[3] and discussed more recently by Schulz[4]. The formulæ derived for the intensity of the transmitted beam may be applied directly to the case of a tooth section of known thickness, if the prisms are assumed to be parallel to each other, by making the thicknesses of the two crystal plates proportional to the numbers of crystallites belonging to the 5° and 40° groups referred to above.

The work forms part of an investigation being carried out for the Dental Disease Committee of the Medical Research Council, and will be published in full elsewhere.

J. THEWLIS.

Physics Department,
National Physical Laboratory,
Teddington.
March 9.

[1] For a survey of this work, see Harders-Steinhäuser, M., *Kolloid Z.* **83**, 86 (1938).
[2] Thewlis, J., *Proc. Roy. Soc.*, B, in the Press.
[3] Fresnel, A., *Ann. Chim. Phys.*, A (2), 17, 167 (1821).
[4] Schulz, H., "Wien-Harms Handb. exp. Physik", Leipzig, **18**, 540 (1928).

Non-Euclidean Geometry in Microscopic Space

CONSIDERATIONS of Heitler, Nordheim and Teller[1], based on Heisenberg's uncertainty principle, show that Newtonian gravitation has only a macroscopic significance; further, in the theory of ultimate particles, it is necessary to introduce a finite length, a, of approximately 10^{-13} cm. (radius of electron or of light nuclei), which is not justified theoretically if we use Euclidean geometry or Einstein's gravitational curvature.

Consequently, it seems more convenient to use a non-Euclidean geometry, but one which is not connected with Einstein's gravitational theory; in the space occupied by the elementary particle, the square of the space-time interval is supposed to be defined by :

$$ds^2 = c^2\left(1 - \frac{r^2}{a^2}\right)dt^2 - \frac{dr^2}{1-r^2/a^2} - r^2(d\theta^2 + \sin^2\theta\, d^2) = c^2\gamma dt^2 - d\sigma^2, \quad (1)$$

which is formally analogous with de Sitter's line element; the internal space is consequently spherical with a curvature $K = 1/a^2$, and the corpuscle is obviously stable, because the Einstein macroscopic relation $R = \varkappa\rho$ between the world-curvature R, the density of matter ρ and the gravitational constant $\varkappa$ (which involves expanding space) is not verified.

Let the line element of the external space be defined by :

$$ds^2 = c^2\left(1 - \frac{a^2}{r^2}\right)dt^2 - \frac{dr^2}{1-a^2/r^2} - r^2(d\theta^2 + \sin^2\theta d^2); \quad (2)$$

therefore, there will be a nuclear attractive field, different from the Einstein or Newtonian field, which can be neglected if a tends to zero.

The spatial part of (1) represents a spherical or elliptic space[2] in which distance is not univocally defined; in the second case, a line element results from the Cayley relation :

$$\sigma = \frac{a}{2i}\log\chi, \quad (i = \sqrt{-1}) \quad (3)$$

where σ is the distance between two points A,B, χ the anharmonic ratio $\overline{AK}/\overline{BK} : \overline{AL}/\overline{BL}$, K and L the points of intersection of $\overline{AB}$ with the absolute quadric[3]. The relation (3) is analogous to Schrödinger formula $S = \frac{h}{2\pi i}\log\psi$ between action function S and wave function ψ; therefore, it is possible to quantize distance; χ is univocal and $\sigma = \sigma_0 + m\pi a$, where m is an integer; the quantum numbers are $\pm\frac{1}{2}$.

Metric notions are completely replaced by projective notions when we introduce for an angle V, Laguerre's formula :

$$V = \frac{1}{2i}\log\varphi \quad (i = \sqrt{-1}); \quad (4)$$

angular quantum numbers are spin quantum numbers $\pm\frac{1}{2}$; we specify that χ and φ have an immediate experimental significance, as ψ in quantum mechanics, while angles and distances lose their actual meaning.

Therefore, metric transformations T, which preserve V, are replaced by corresponding homographic transformations H which preserve the anharmonic ratio $\varphi = \frac{\lambda_3 - \lambda_1}{\lambda_3 - \lambda_2} \cdot \frac{\lambda - \lambda_2}{\lambda - \lambda_1}$; the projective parameter λ is changed by H in

$$\lambda' = \frac{\alpha\lambda + \beta}{\gamma\lambda + \delta}; \quad (5)$$

it is easy to prove that, in the usual cases, this transformation is identical with the unitary substitution of two variables of Van der Waerden[4]

JEAN MARIANI.

Laboratoire de Physique Théorique,
Collège de France,
Paris.
March 15.

[1] From Gamow, *Phys. Z.*, **38**, 800–814 (1937).
[2] Tolman, "Relativity, Thermodynamics and Cosmology" (Oxford, 1934), p. 346.
[3] Von Laue, "La Théorie de la Relativité" (G. Villars, 1926), **2**, 130.
[4] Garnier, "Leçons d'Algèbre et d'Analyse" (G. Villars, 1936), **2**, 90, 135.

Temporary Poikilothermy in Birds

IT appears to be well known to aviculturists and collectors that humming-birds are capable of a physiological state somewhat resembling hibernation in mammals, but of short duration; however, the fact seems not to have been recorded in the scientific literature. We propose to refer to this peculiar condition as 'torpidity'. The observations here recorded were made partly in the field in Ecuador, partly in the Zoological Society's Gardens at Regent's Park.

In complete torpidity, the birds are quite rigid, with head drawn back and pointed up, flattened plumage and closed eyes, and exhibit no perceptible respiratory movements. The condition is reversible, recovery in captivity taking 10–15 minutes, at high altitudes and low temperatures in Ecuador up to 30–35 minutes.

Partial torpidity is also observed. In captive birds this is characterized by violent breathing, ruffled plumage and the loss of the power of flight. Recovery from this state takes 4–5 minutes. Partially or completely torpid birds on being touched emit a peculiar expiratory whistle, which, it is suggested, may be due to deflation of the air-sacs.

Torpidity is promoted by numerous factors : (1) low temperature ; (2) lack of food ; (3) poor condition ; (4) a diurnal effect, birds being much more susceptible at night ; (5) a seasonal rhythm, captive birds having proved much more susceptible in

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